

# Applications of Magnetic Field Mapping in MHD Analysis Path, Surface and Volume Integration Techniques

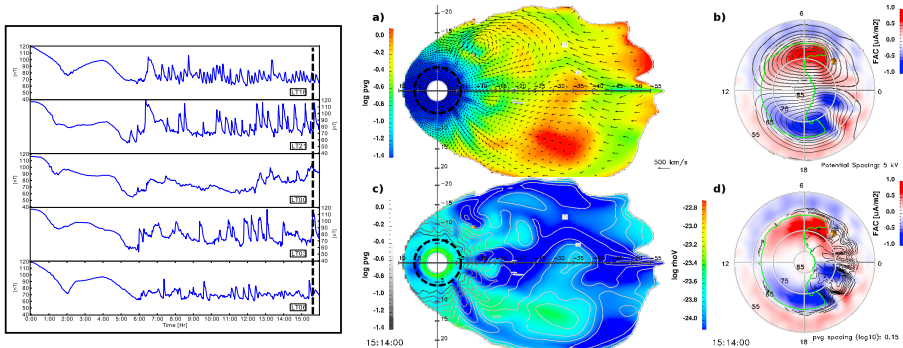
Asher Pembroke<sup>1</sup>, Frank Toffoletto<sup>1</sup>, Michael Hesse<sup>2</sup>

<sup>1</sup>Rice University, Houston, <sup>2</sup>CCMC, Goddard Space Flight Center

GEM 2011 Magnetic Mapping WG June 30, 2011

# Why MHD modelers need mapping

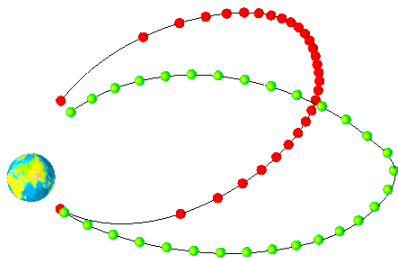
Several physical processes of global MHD models require magnetic field mapping between domains, such as the dynamics of flux tube entropy and mass content.



Magnetic Field Signatures of LFM-RCM-MIX at geosynchronous orbit (left). Field line entropy (a), density (c) in LFM equatorial plane. (b) Birkeland currents and ionospheric potential. (d) Birkeland currents and entropy contours.

## What's missing with this picture?

## “New” Technique: Field line interpolation (1-D)



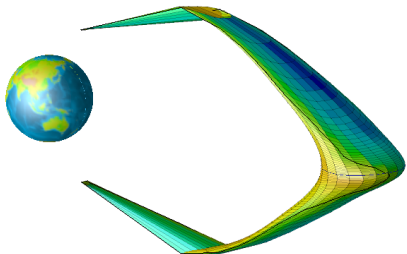
Field lines with resolution lowered to just 30 vertices each. Basic shape can still be captured.

### Field line reduction

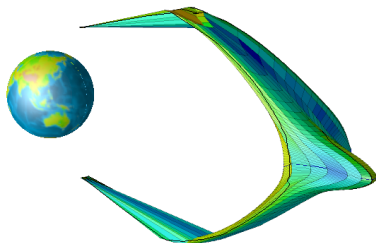
- Given  $\vec{F}(\vec{r})$ , can re-sample by fixed partially integrated  $\int ds/F$ .
- Drastically reduces storage requirements for each trace.
- Flexible weighting schemes: Partial flux tube volume ( $\int_0^s ds/B$ ) or fixed length ( $\int_0^s ds$ ).
- **Protip: Store partial integrals for Mass Content, Entropy, Flux Tube Volume, and Local MHD 14-Vector**

## Sampling Permits Flux Tube Construction (2-D application)

Surfaces can be built by extrusion along sampled field lines.

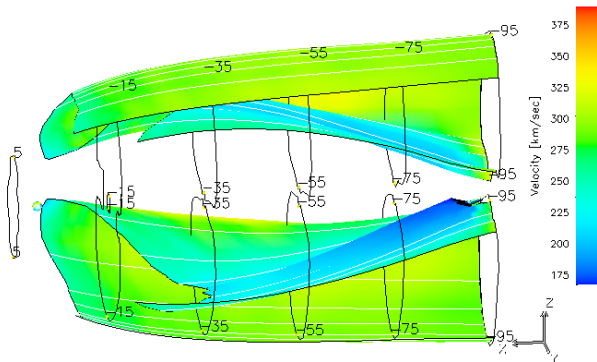


Flux tube mapped from a  $1R_e$  circle in the equator (LFM)



The same flux tube, 8 minutes later (based on streakline trace).

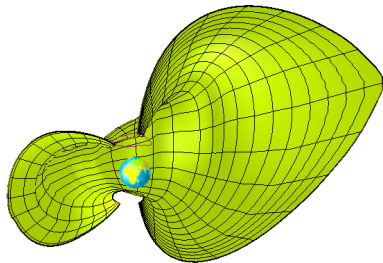
## Technique also applies to flow surfaces



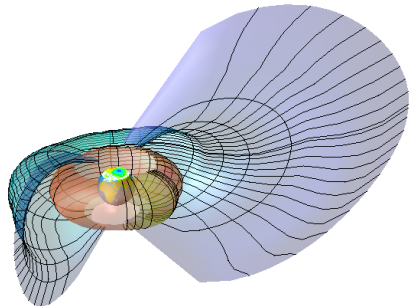
Construction of a plasma mantle drift surface: Resample streamlines at constant  $\Delta t$ ; extrude from a  $\beta = 1$  curve in the distant tail ( $-95R_E$ )

## 3D version → Complete Magnetic Mapping

Extrude volume elements from surface connections (i.e. ionospheric mesh/equatorial plane). Previous surfaces are now implicit (isosurfaces).



Equipotential drift surface

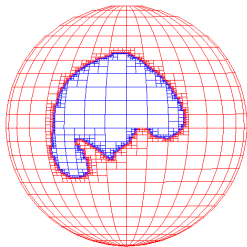


Adiabatic drift shells

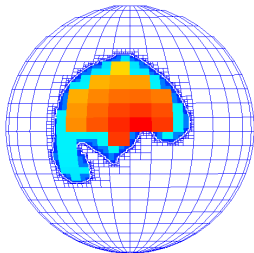
$$v_D = \frac{\vec{B}}{B^2} \times \nabla \left( \frac{\lambda V^{-2/3}}{q} + \Phi \right)$$

# 3D Global Separatrix: Start with Local Topology Search

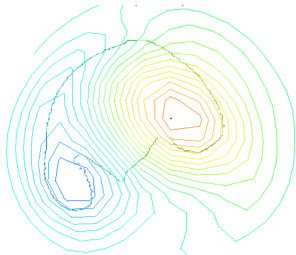
Multi-resolution Separatrix search inspired by Warren & Schaefer's scheme for subdivision surfaces [2]



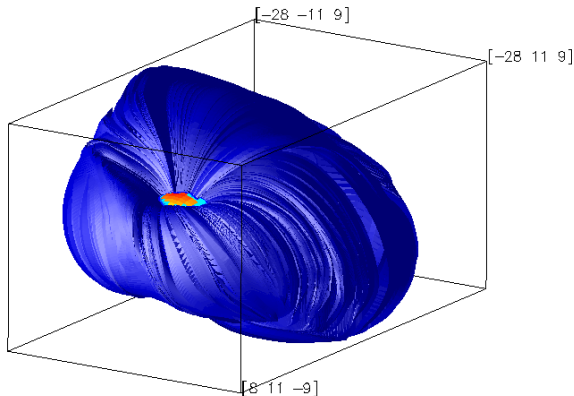
Fluxes computed per cell on the starting mesh



Reconnection potential can be calculated from  $V_{max} - V_{min}$  along the separatrix



## 3D Global Separatrix: ... Build the Field Line Volume



3D separatrix surface, interpolated from 2602 field lines forming a volume. Compare with brute force volume search (61k).

Bounding box gives us a measure of both the geoeffective length and magnetopause location!



## Yeah that's pretty, but what *else* can we do with it?

Magnetic Mapping *is* Measurement!

- (1) Build a frame out of existing field line data (the bottleneck)
- (2) Populate it with global physical measurements - your field integrations, topologies, ionospheric quantities, etc.

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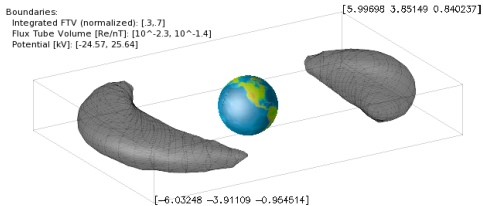
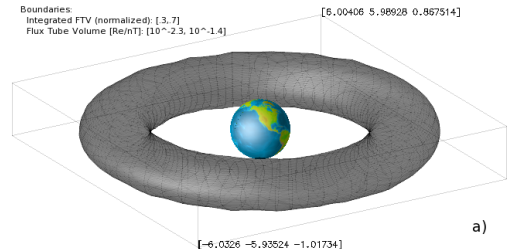
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- (5) Make bold statements about the physics captured in your model! (optional)

# Surface Construction: ROI Boundary



- 1 Reparameterize volume with user-defined boundaries:

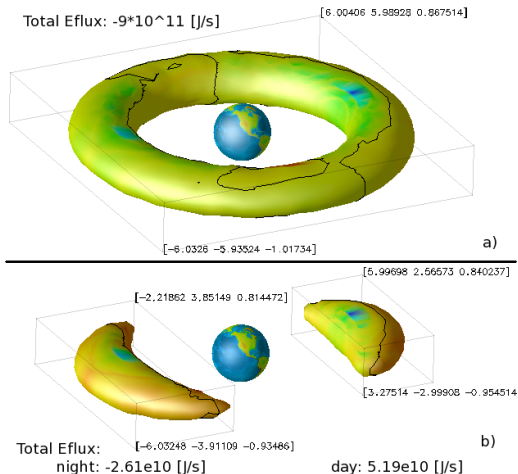
$$f'_i = 2 \left| \frac{f_i - f_{i,\min}}{f_{i,\max} - f_{i,\min}} - .5 \right|$$

- 2 Combine parameterizations with a single shape function, i.e. spherical:

$$F_{ROI} = \sqrt{\sum_i f_i'^2} - 1$$

- 3 Compute isosurface given by  $F_{ROI} == 0$

# Flux Integration



- 1 Interpolate flux quantities at surface locations, e.g.:

$$F_e = (U + P) \vec{v} + \vec{E} \times \vec{B}$$

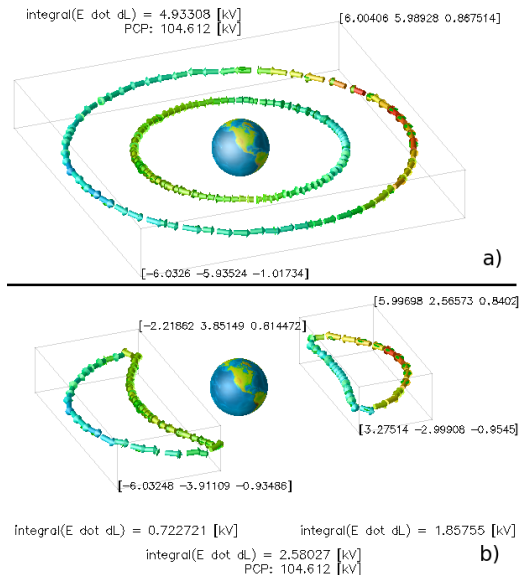
(computed during surface construction)

- 2 Integrate over surface to obtain time rate of change for the volume, i.e. for energy:

$$\frac{\partial U_{tot}}{\partial t} = - \int_S F_e(S) \cdot \vec{d}\vec{a}$$

Surface normals must point away from the ROI!

# Path Integration



Path integrations can be simplified via surface fluxes:

- 1 Calculate fluxes per cell along surface, e.g.

$$d\phi = \vec{B} \cdot \vec{da}$$

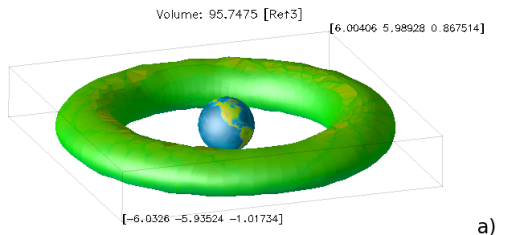
- 2 Compute the bounding curve of  $\pm$  flux, e.g.

$$\vec{dl} \equiv \delta(d\phi > 0)$$

- 3 Integrate over the path, e.g.

$$\frac{\partial \Phi}{\partial t} = - \oint \vec{E} \cdot \vec{dl}$$

# Volume Integration



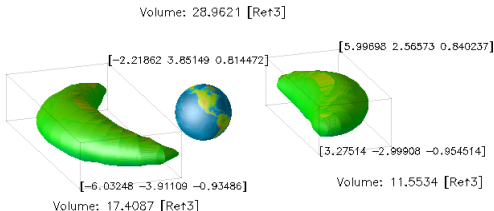
We may wish to integrate  $f_i$  ( $\rho, P, B^2$ ) over the RIO:

$$f_{tot} = \int_{ROI} f_i dv = \int f_i \vec{ds} \cdot \vec{da}$$

Compute the field line integrals:

$$f_L(s) \equiv \int_0^s \frac{f_i(s)}{B(s)} ds$$

Note that  $\nabla \cdot (f_L \vec{B}) = B \cdot \nabla f_L + f_L \nabla \cdot \vec{B} = f_i(s)$   
Apply divergence theorem:



b)

$$\int_{ROI} \nabla \cdot (f_L \vec{B}) = \oint_S f_L \vec{B} \cdot \vec{da}$$

$$\therefore f_{tot} = \oint_S f_L(s) d\phi$$



## Possible applications

Flexible ROI boundaries allow for...

- How far does a field line move in a minute? In MHD: how fast does a flux region evolve?
- investigation of Field-Aligned Current structure and origin
- quantifying the volume, energy, and mass of the closed field region
- determining the mass and energy available to drift paths
- the total forces acting on a ROI and the relative forces
- the magnetic flux into the ROI ( $\int E \cdot \vec{dl}$ )
- the integrated vorticity of the flow ( $\int V \cdot \vec{dl}$ )

## Proceed with caution...

When tracking a measurement with respect to time:

- Use streaklines or ROI boundaries we know are conserved with the flow.
- Beware when applying the maxwell-faraday equation: it only works when the integration surface is tied to the flow!
- Volume integration only works for divergenceless fields...

## Implementation

- A set of macros/networks built and prototyped in CISM-DX (Open-DX)
- Project hosted on google code: <http://code.google.com/p/cismdxalgs/>
- Repository has video tutorials and a rough draft of a paper in progress:  
A General Approach to Magnetospheric Visualization
- CISM-DX is only the beginning...

## Delivery to the Community Coordinated Modeling Center

Integration with the CCMC framework will occur over the coming months.

- CCMC's Kameleon-plus (C++ library) field line tracer, compatible with most CCMC models
- We recently extended the field line class to include integration and sampling.
- Need to implement flexible mesh data structure and isosurface generator (see Dual Contouring [1]).
- Development of sanity checks are crucial: compare with direct volume integrations.



Future development:

- How do we adapt the approach to other Heliospheric regions?
- Can global measurement aid data-model comparisons?

## Acknowledgements

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## References

-  Tao Ju, Frank Losasso, Scott Schaefer, and Joe Warren, *Dual contouring of hermite data*, SIGGRAPH '02: Proceedings of the 29th annual conference on Computer graphics and interactive techniques (New York, NY, USA), ACM, 2002, pp. 339–346.
-  Joe Warren and Scott Schaefer, *A factored approach to subdivision surfaces*, IEEE Comput. Graph. Appl. **24** (2004), no. 3, 74–81.