Entropy and Earthward Transport in the Plasma Sheet

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(Thanks to Colby Lemon and Jichun Zhang for use of unpublished results)

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2. Basic Theory of Plasma Sheet Entropy

Drift Motion on Closed Field Lines With Slow Flow

- Assume that the energy in drift motion is small compared to thermal energy (bounce and gyro motion), which allows use of the bounce-averaged-drift approximation.
- It is elegant to use Euler potentials (**B**=$\nabla\alpha\times\nabla\beta$) to label the field lines and the drift motion of the particles [*Stern*, *Am.J.Phys.*, *38*, 494, 1970], in which case the bounce-averaged-drift equations become simply

$$\frac{d\alpha}{dt}=\frac{1}{q}\frac{\partial H}{\partial\beta},\ \frac{d\beta}{dt}=-\frac{1}{q}\frac{\partial H}{\partial\alpha} \tag{1}$$

- These equations are equivalent to the equations for incompressible fluid flow in 2D. ($H\rightarrow$stream function).
- Different useful approximations can be obtained by using different types of expressions for *H*.

Kinetic Theory Definition of Entropy

- Definition of entropy:

$$S = -\int d^3x \int d^3p \; f \ln(f) = -H_B V \qquad (2)$$

where

$$N = \int d^3x \int d^3p \; f \qquad (3)$$

H_B = Boltzmann H function, and f = distribution function (e.g., *Liboff*, *Kinetic Theory*, Sect. 3.3.7).

- If the particle motion is Hamiltonian, then f is conserved along a path in phase space, and the volume of phase space occupied by a set of particles on nearby trajectories is also conserved.
- If particle motion is determined by large-scale **E** and **B**, then the motion is Hamiltonian.
  - Collisional and dissipational processes typically make the motion non-Hamiltonian.

Entropy of a Perfect Gas

$$S = -\int d^3x \int d^3p \; f \ln(f) \qquad (2)$$

$$N = \int d^3x \int d^3p \; f \qquad (3)$$

- Isotropic perfect gas:

$$f = \frac{n}{W_o^{\;3/2}} \, g\left(\frac{W}{W_o}\right) \qquad (4)$$

where n=number density, W=particle energy, n=number density, W_o=average energy, g = arbitrary function. Substituting (4) in (2) and (3) and using perfect gas law gives, for the entropy per particle

$$\frac{S}{N} = \frac{3}{2}\ln\left(\frac{P}{n^{5/3}}\right) + \Lambda \qquad (5)$$

where Λ depends just on the shape of the distribution function.

- For Hamiltonian drifts, $P/n^{5/3}$ is conserved, provided that the volume of phase space occupied by the gas remains compact, so that you can relate what happens at (x, t) to what happened at (x', t').

Plasma With Isotropic Pressure and Frozen-in Flux

$$\frac{S}{N} = \frac{3}{2}\ln\left(\frac{P}{n^{5/3}}\right) + \Lambda \tag{5}$$

- The easiest way to ensure that the distribution function remains compact is to take a Hamiltonian of the form

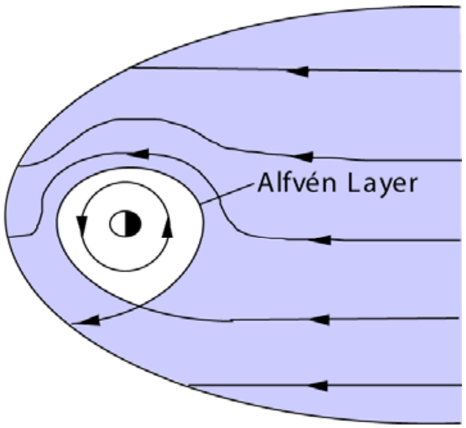
$$H = q\Phi(\alpha, \beta, t) \tag{6}$$

- Then the particle motion is **E**x**B** drift and $E_{||}$=0. That ensures frozen-in flux, which ensures the conservation of N=nV = particles per unit magnetic flux, where n=number density, and V=$\int ds/B$ is the volume of a tube of unit flux.

$$\frac{S}{N} = \frac{3}{2}\ln\left(PV^{5/3}\right) - \frac{5}{2}\ln\left(N\right) + \Lambda \tag{7}$$

- Thus Hamiltonian motion implies conservation of $PV^{5/3}$ along a drift path assuming isotropic pressure, frozen-in flux, no loss of particles from the flux tube, and also that the shape of the distribution function remains constant.

Application to the Magnetosphere



- Shaded region is one in which the flow is slow compared to MHD waves speeds.
- Ideal MHD case:
  - If $PV^{5/3}$ is uniform on the portions of the outer boundary where there is inflow, and independent of time, then $PV^{5/3}$ is uniform throughout the slow-flow region, except for the trapped-particle region.

Plasma With Isotropic Pressure and Particles That Gradient/Curvature Drift in Addition to ExB Drift

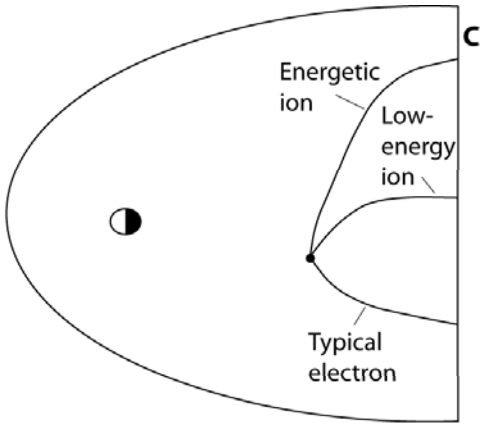
$$\frac{d\alpha}{dt} = \frac{1}{q}\frac{\partial H}{\partial \beta}, \quad \frac{d\beta}{dt} = -\frac{1}{q}\frac{\partial H}{\partial \alpha} \qquad (1)$$

- The corresponding Hamiltonian is

$$H = q\Phi(\alpha,\beta,t) + \lambda V(\alpha,\beta,t)^{-2/3} \qquad (8)$$

where λ = energy invariant. Substituting (8) in the drift equations (1) gives the standard equations for bounce-averaged ExB and gradient-curvature drift (e.g., *Harel et al.*, *JGR*, *86*, 2217, 1981) for isotropic pressure.

- Since now different particles drift at different velocities, the full region of phase space occupied by the plasma doesn't remain compact as the particles drift.
- The full S/N of the plasma is not conserved.
- It is useful to consider different regions of phase space separately.

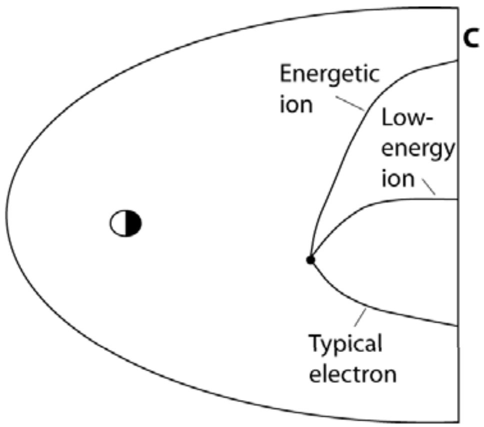


Plasma With Isotropic Pressure and Particles That Gradient/Curvature Drift in Addition to ExB Drift

- Divide the energy distribution up by chemical species and λ levels. Within each level $PV^{5/3}$ is conserved :

$$P(\mathbf{x},t)V(\mathbf{x},t)^{5/3} = \sum_s P_s\left[\mathbf{x}_{Cs}(\mathbf{x},t),t_{Cs}(\mathbf{x},t)\right]V\left[\mathbf{x}_{Cs}(\mathbf{x},t),t_{Cs}(\mathbf{x},t)\right]^{5/3} \qquad (9)$$

  - If the distribution function is uniform on the portion of the boundary where particles are entering the region, $PV^{5/3}$ is uniform throughout the sheet.
    - Except for the contribution of particles that are on trapped orbits.



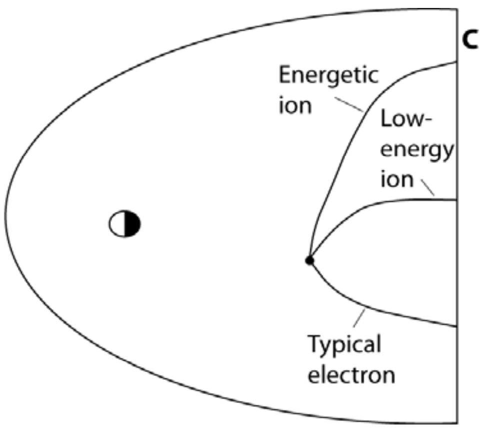
Plasma Consisting of Particles That Drift Conserving First Two Adiabatic Invariants

$$\frac{d\alpha}{dt} = \frac{1}{q}\frac{\partial H}{\partial \beta}, \frac{d\beta}{dt} = -\frac{1}{q}\frac{\partial H}{\partial \alpha} \qquad (1)$$

- The corresponding bounce-averaged-drift Hamiltonian is

$$H = q\Phi(\alpha,\beta,t) + W_K(\mu,J,\alpha,\beta,t) \qquad (10)$$

  where W_K is the kinetic energy in the particle's bounce and gyro motion, which usually has to be calculated numerically.
- Since this drift motion is Hamiltonian, *f* should again be conserved along a drift path.
- Entropy is still defined and conserved, but it isn't simply related to pressure or energy density.
- If we still use $PV^{5/3}$ conservation to estimate the total particle energy on a flux tube, do we underestimate or overestimate the adiabatic energization that accompanies earthward transport?
  – Answer: We underestimate it [*Wolf et al.*, *JGR*, *104*, A00D05, 2009].



Concluding Comment on Entropy

- I will probably sometimes refer to $PV^{5/3}$ as "entropy", but remember that

$$\frac{S}{N} = \frac{3}{2}\ln\left(PV^{5/3}\right) - \frac{5}{2}\ln\left(N\right) + \Lambda \qquad (7)$$

- The entropy of particles in a small region of phase space is conserved under Hamiltonian motion.
- $PV^{5/3}$ is conserved exactly only in cases where
  - Frozen-in-flux approximation is valid
  - Pressure is isotropic
  - Shape of energy dependence of distribution function is conserved in the drift
- Constraints on $PV^{5/3}$ are weaker and more complicated when one or more of these assumptions are violated.

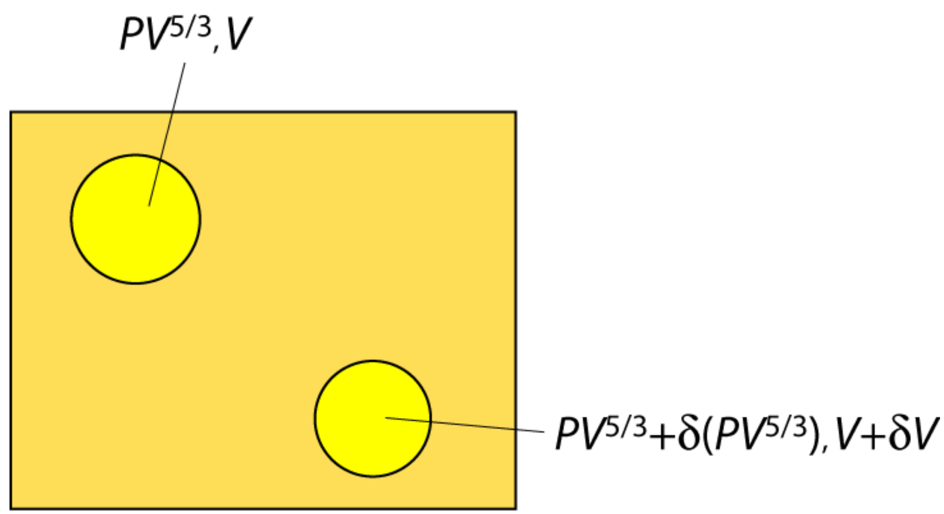
3. Interchange Instability

Interchange Instability Criterion – Ideal MHD, low β

- Standard textbook criterion for interchange instability (*Schmidt*, *Physics of High Temperature Plasmas*, 2nd ed., 1979):

- Exchange flux tubes with equal magnetic flux.
- Assuming adiabatic compression, potential energy decreases under the exchange if

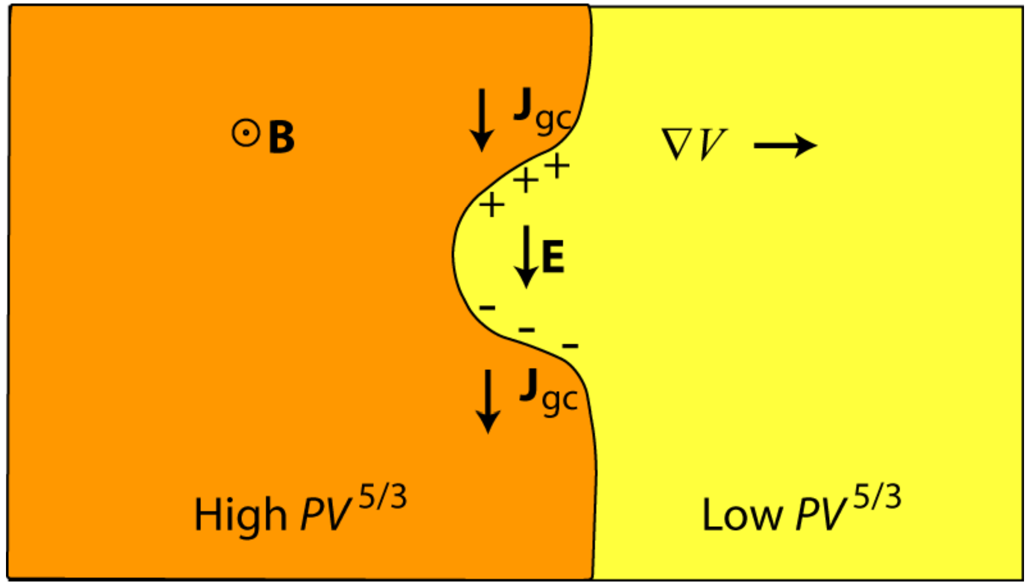
$$\delta\left(PV^{5/3}\right)\delta V < 0$$



- This analysis assumes that the magnetic field does not vary in the interchange.
  - Just considers the change in particle thermal energy.
  - This simple interchange is most meaningful for a low-β plasma.

Intuitive Picture of Interchange Instability

$$\mathbf{v}_{GC} = \frac{\mathbf{B} \times \nabla\left(\lambda V^{-2/3}\right)}{qB^2}$$



- Picture shows situation where higher-$PV^{5/3}$ flux tubes are nearer the Earth, on lower-volume flux tubes.
- The divergence of gradient/curvature-drift current produces charges on the sides of the ripple
  - Produces an E field that causes the ripple to grow→instability.
- The system is unstable if $PV^{5/3}$ decreases in the direction of increasing V. See *Xing and Wolf* (*JGR*, *112*, A12209, 2007) for details.

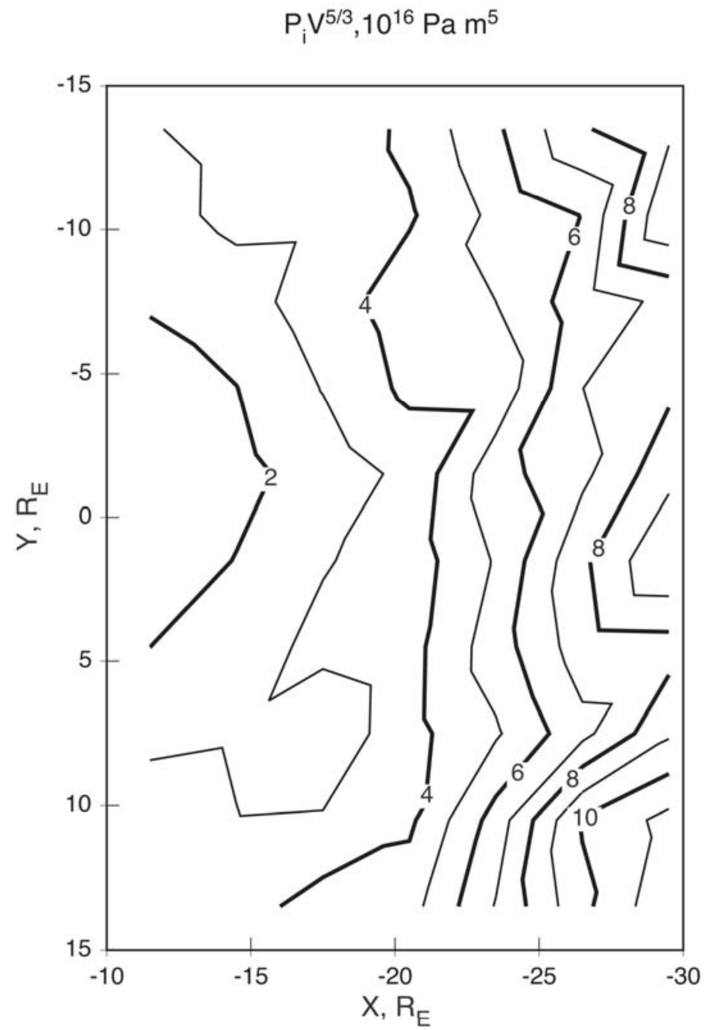
4. Statistical Plasma-Sheet Models and Pressure-Balance Inconsistency

Entropy in the Statistical Plasma Sheet

- *Borovsky et al.* (*JGR*, *103*, 20297, 1998) showed that nV increases with geocentric distance and examined the mild increase of $P/n^{5/3}$ with distance.
- *Garner et al.* (*JGR*, *108* (A8), 2003) combined the T89 B-field model and *Paterson et al*. (*JGR*, *103*, 11811, 1998) to make contour maps of equatorial $PV^{5/3}$.
- *Xing and Wolf* (*JGR*, *112*, 12209, 2007) combined T96 B-field model with Tsyganenko-Mukai (*JGR*, *108*(A3), 2003) plasma sheet to make contour maps of equatorial $PV^{5/3}$.
- *Kaufmann et al*. (*JGR*, *114*, A00D04, 2009) obtained equatorial contour maps of $PV^{5/3}$ using Geotail data to obtain both pressures and magnetic fields.
- *Wang et al*. (*JGR*, *114*, A00D02, 2009) computed partial entropies for different invariant energies λ and also used the statistical flow velocities to determine whether the partial entropy is constant along a drift path.

Pressure Balance Inconsistency

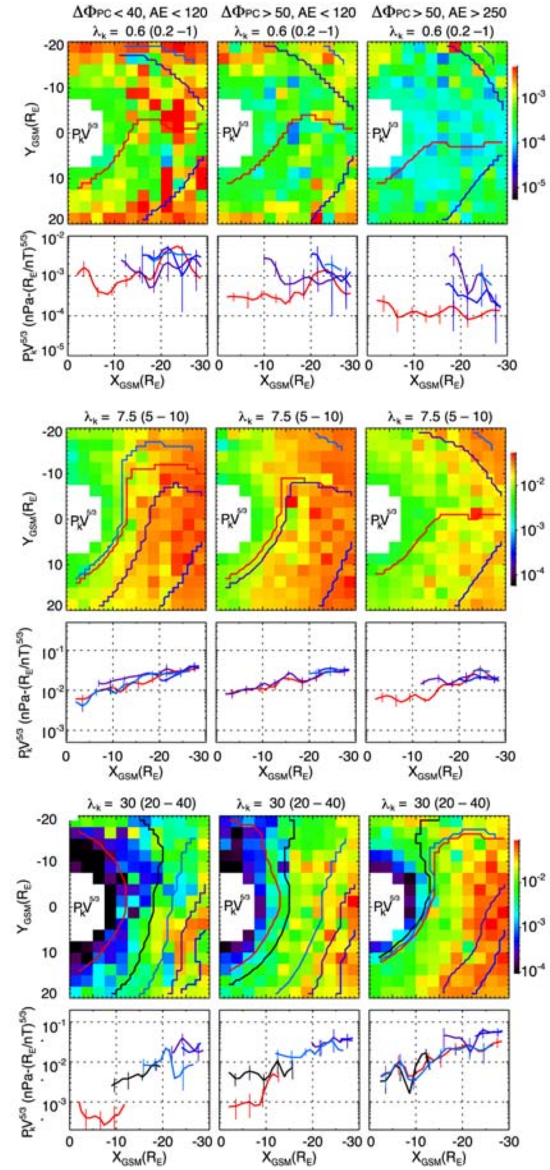
- All of these statistical analyses agree that entropy increases with geocentric distance, which is roughly the direction of $\nabla\Phi$.
- The inconsistency between the theoretical expectation that $PV^{5/3}$ should be roughly constant in the plasma sheet and the fact that $PV^{5/3}$ increases downtail in statistical models is called the "pressure balance inconsistency" or sometimes the "pressure crisis" (*Erickson and Wolf*, *GRL*, 897, 1980).
- "Entropy inconsistency" would have been a better name.
- $P/n^{5/3}$ is much more uniform than $PV^{5/3}$. It's not that the flux tubes have gotten adiabatically cooled as they move earthward. They've lost particles (Borovsky, Kaufmann).



(*Kaufmann et al.*, *JGR*, *109*, A08204, 2004)

Conservation of Partial Entropy Along Drift Paths

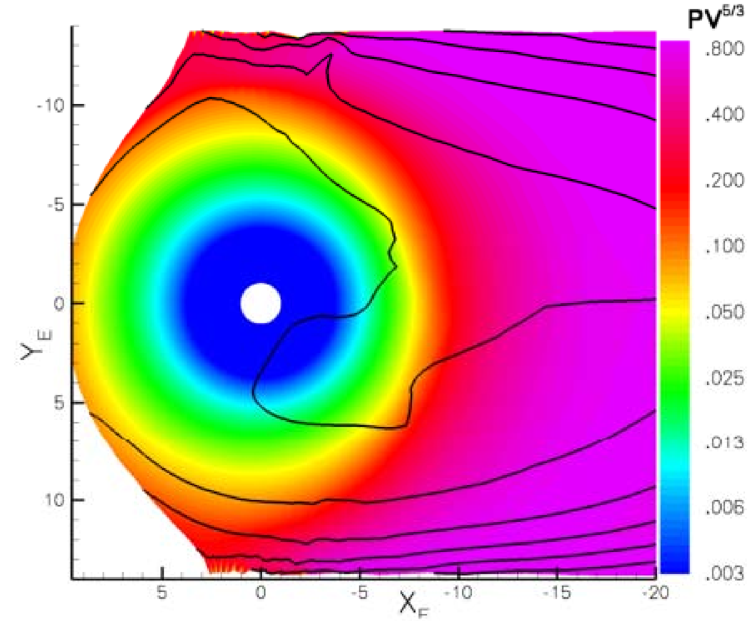
- *Wang et al.* (*JGR*, *114*, A00D02, 2009) investigated the conservation of partial entropy (for given energy invariant λ_s), along drift paths, which were computed assuming electric fields estimated from the average flow velocities.
- The paper interprets the deviations from constancy of the partial entropy along drift paths as insignificant.

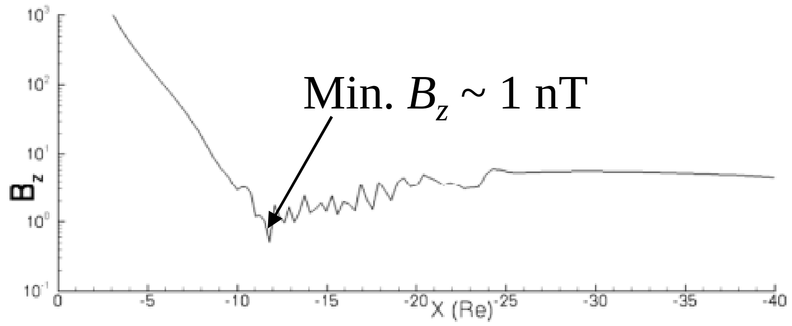


5. Entropy-Conserving Self-Consistent Solutions

Strong, Steady Adiabatic Convection

- When we enforce strong convection for hours in the RCM-E code, which keeps recalculating the magnetic field to keep it in approximate force balance with the RCM-computed $PV^{5/3}$ values, we always get a configuration that is highly stretched in the inner plasma sheet.
- $PV^{5/3}$ was assumed to be uniform on a boundary out in the tail.
- Note that $PV^{5/3}$ is nearly uniform beyond ~ 10 R_E, in contrast to statistical models.
- When we run RCM-E for a long time with a strong potential drop, the configuration reaches a highly stretched configuration
  - Nothing like a substorm expansion
  - No injection into the ring current

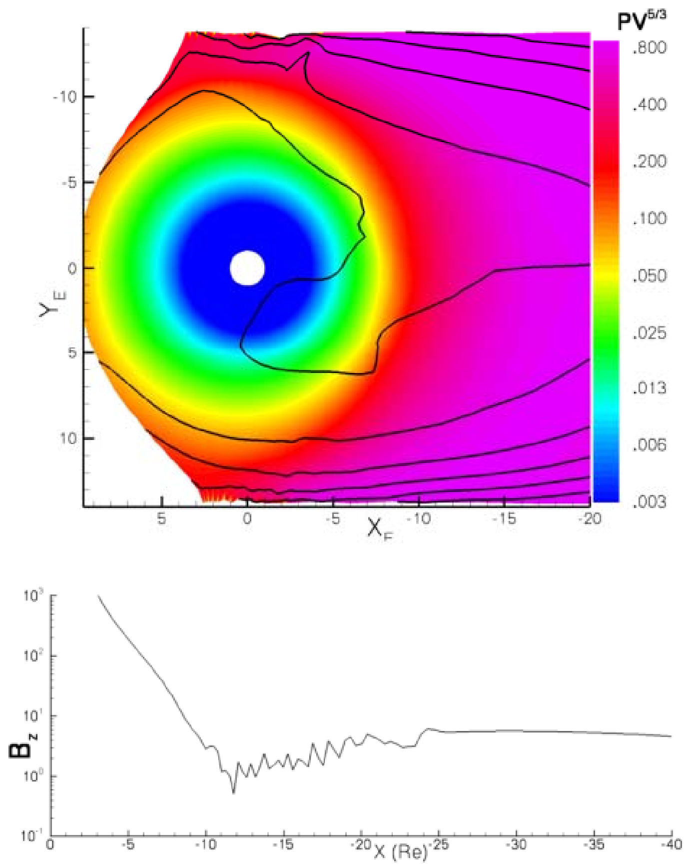




120 kV potential drop.
C. Lemon (*Ph.D. thesis*, Rice, 2005)

Pressure-Balance Inconsistency

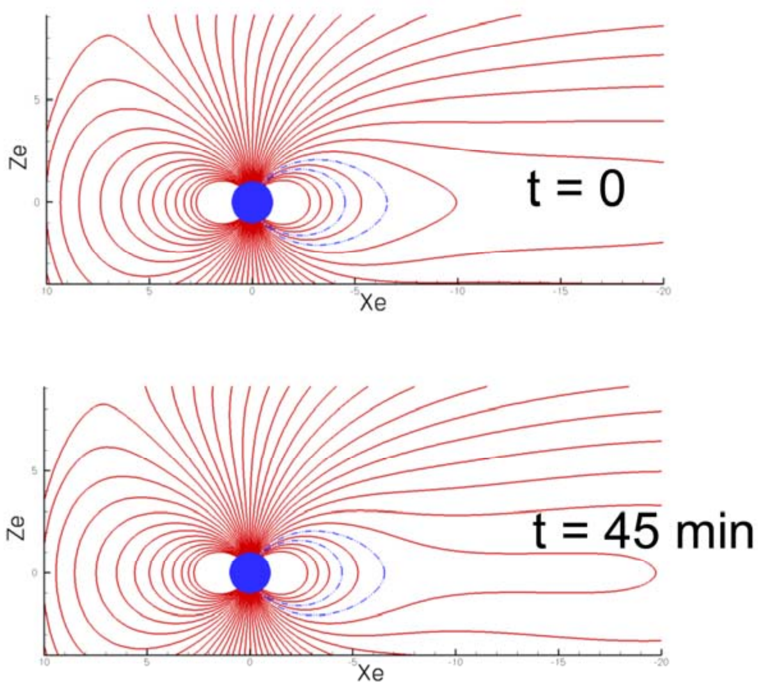
- The equatorial particle pressure decreases slowly downtail.
- In statistical models, the flux tube volume increases more rapidly.
- Thus $PV^{5/3}$ normally increases downtail.
- The RCM-E model avoids this in a steady convection configuration like the one shown to the right by creating a deep minimum in equatorial field strength in the inner plasma sheet (~ 1 nT). The inner plasma sheet flux tubes grow bigger, allowing volume to increase downtail only slowly.
- Qualitatively similar results: *Hau* (*JGR*, *96*, 5591, 1991), *Erickson* (*JGR*, *97*, 6505, 1992), *Toffoletto et al.* (*ICS-3 Proceedings*, 1996), *Lemon et al*.(*GRL*, *31*, L21801, 2004), *Wang et al.* (*JGR*, *109*, A12202, 2004).



(*Lemon*, Ph.D. thesis, Rice, 2005)

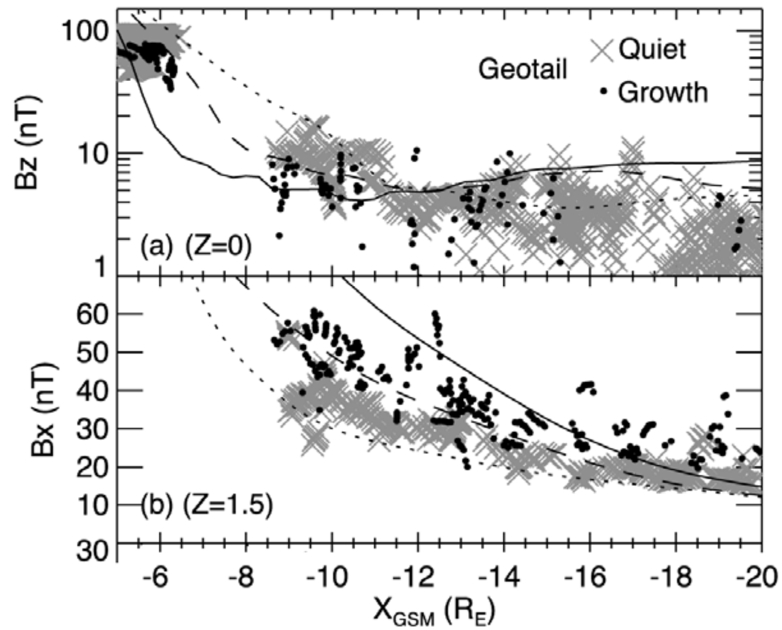
Connection to Substorm Growth Phase

- Our group's interpretation of the highly stretched configuration is that steady adiabatic convection from out in the tail naturally produces a highly stretched inner plasma sheet.
  - Tail lobe field also strengthens.
  - Increased energy stored in the tail.
- Resembles a substorm growth phase.
- Natural interpretation is that, if the physical configuration gets stretched enough, then some instability gets triggered, which causes violation of the adiabatic condition and reduces entropy on some flux tubes (*Toffoletto et al.*, *Proc.ICS-5*, 2000)



(C. Lemon, Ph. D. thesis, Rice, 2005)

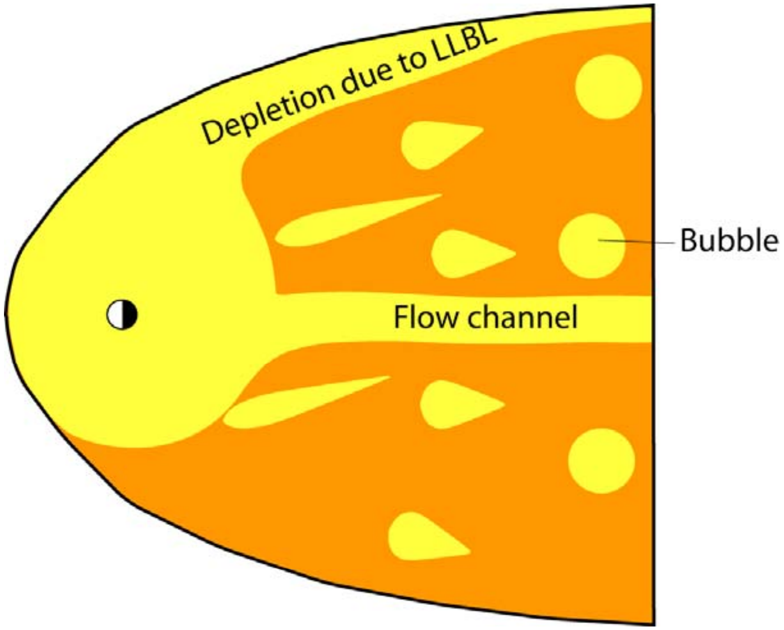
Different Calculation



- Using a somewhat similar procedure, *Wang et al.* (*JGR*, *109*, A12202, 2004) get somewhat less stretched configurations.
  - They never get configurations as stretched as the one from *Lemon* (2005).
- Differences in the *Wang* and *Lemon* calculations:
  - *Wang et al*. adopted a boundary condition with cool ions on the dusk side and allowed flow in through that bounary. Lemon enforced weak ExB drift out through the flanks.
  - While *Lemon* used a friction-code equilibrium solver to get a full 3D equilibrium, *Wang et al.* modified a T96 model to achieve equilibrium in the xz plane.
  - *Lemon* used an RCM-computed self-consistent potential electric field, while *Wang et al*. use an MSM-based assumed potential.
- Wang calculation still exhibits the pressure balance inconsistency, because its inner plasma sheet is more stretched than Tsyganenko models, but the stretching is not as extreme as in RCM-E calculation, and it agrees with growth-phase data.

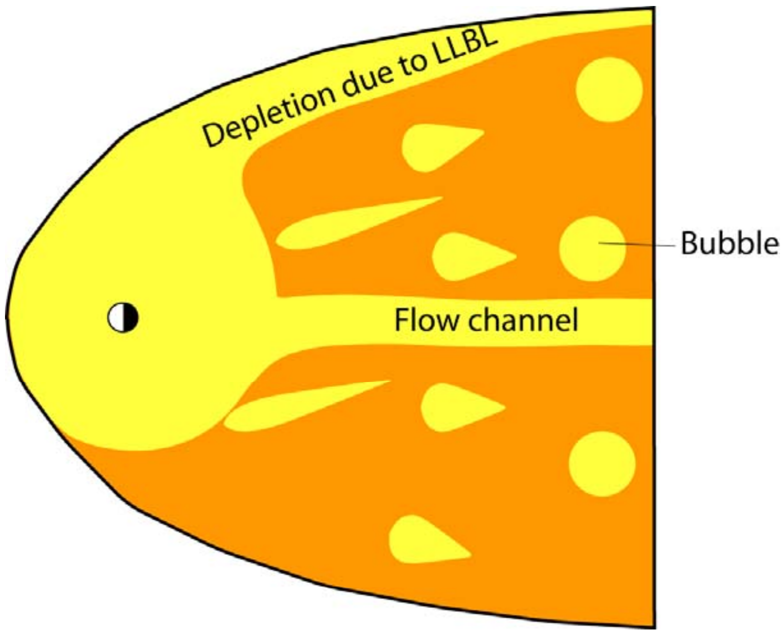
Resolution of the Pressure Balance Inconsistency

- Two main mechanisms have been suggested:
  1. Gradient/curvature drift. If the dawnside LLBL produces ion population with $PV^{5/3}$ much lower than in the distant tail, and a lot of the inner plasma sheet comes from the LLBL, then the inner plasma will have smaller $PV^{5/3}$ than the middle and distant sheet.
     - Development of idea: *Tsyganenko* (*Planet. Space Sci.*, *30*, 1007, 1982), *Kivelson and Spence* (*GRL*, *15*, 1541, 1988), *Spence and Kivelson* (*JGR*, *98*, 15487, 1993), *Wang et al*. (*GRL*, 29 (24), 2002; *JGR*, *109*, A12202, 2004; *JGR*, *114*, A00D02, 2009), *Lyons et al*. (*JGR*, *114*, A00D01, 2009).
     - Works best for periods of slow convection.



Resolution of the Pressure Balance Inconsistency

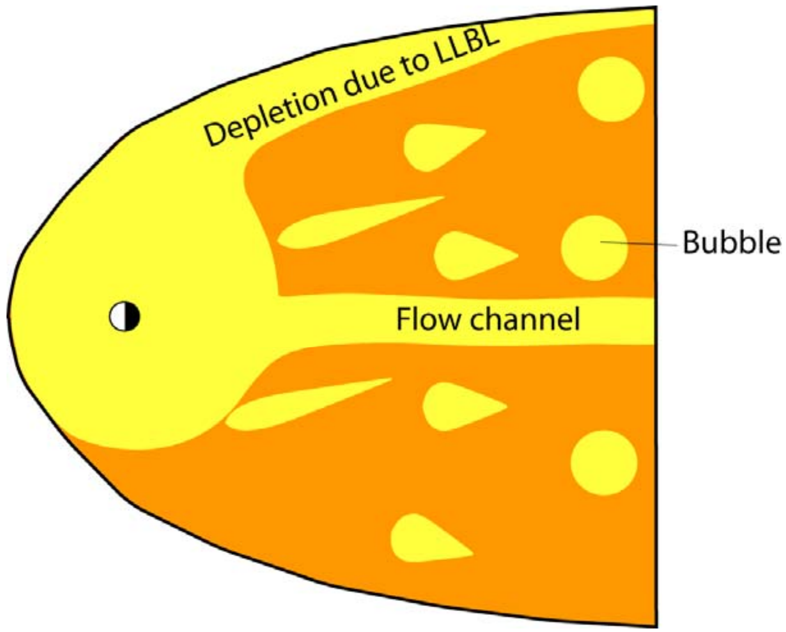
2. Flow channels and bubbles with reduced $PV^{5/3}$ and strong earthward flow.
   - Development of idea: *Sergeev and Lennartsson* (*PSS, 36*, 353, 1988), *Sergeev et al.* (*PSS*, *38*, 355, 1990), *Pontius and Wolf* (*GRL*, *17*, 49, 1990), *Chen and Wolf* (*JGR*, *98*, 21409, 1993; *JGR*, *104*, 14613, 1999).
   - Both BBFs and substorm expansions produce bubbles.



- The bubbles and flow channels transport low-$PV^{5/3}$ flux tubes to the inner plasma sheet.
- If bubbles (BBFs) and flow channels, combined with the gradient/curvature-drift effect, provide enough low-$PV^{5/3}$ flux tubes to the inner plasma sheet that the magnetic field doesn't stretch to the breaking point, you get a Steady Magnetospheric Convection event.
  - If they don't, then you get a substorm expansion.

Resolution of the Pressure Balance Inconsistency

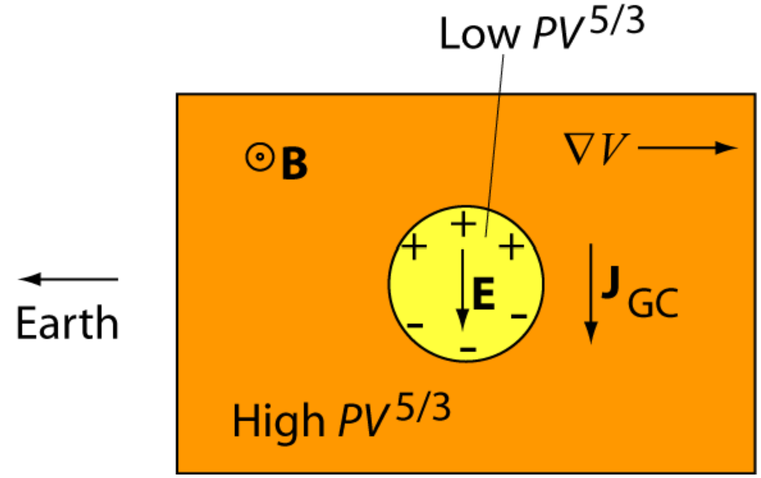
- Both gradient/curvature-drift and bubble mechanisms operate.
- They are not mutually exclusive.
  - In fact, the combination should be stronger than the sum of its parts.
  - If a large fraction of the total transport is in flow channels, bubbles, then the electric field across the orange (high-$PV^{5/3}$, slow flow) channels decreases, which makes gradient/curvature drift a more efficient loss mechanism.
- There is evidence of flow channels coming in from flanks, where V and thus $PV^{5/3}$ are small (*Peroomian*, *Lu*)
  - We need to simulate cases like that.



6. Bubbles in the Plasma Sheet and Creation Mechanisms

Bubbles in the Plasma Sheet

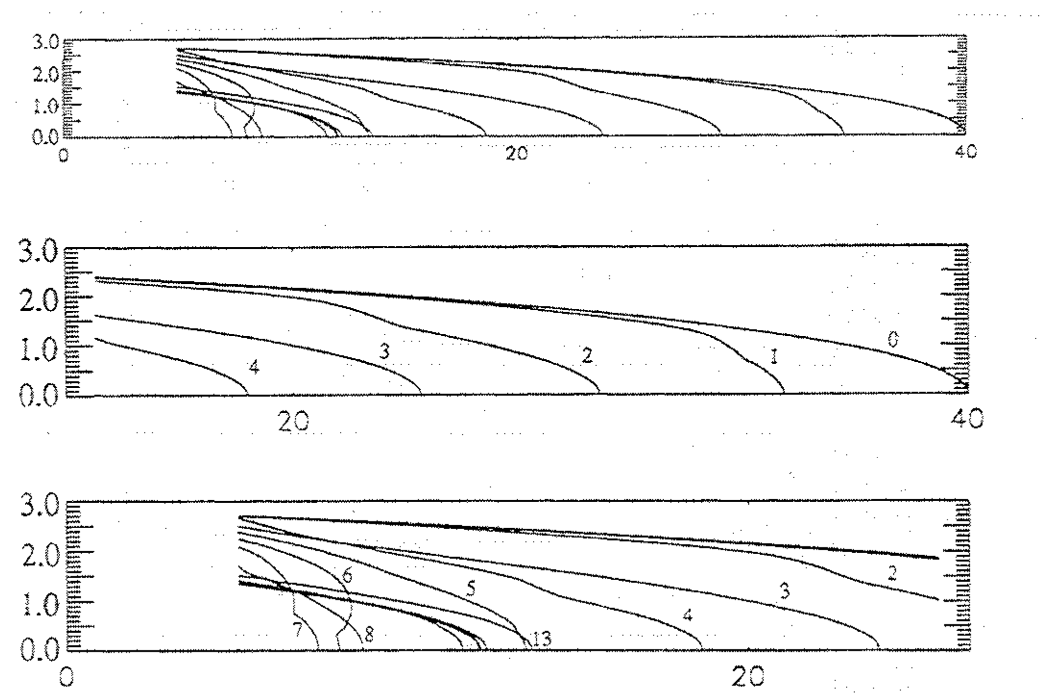
- *Pontius and Wolf* (*GRL*, *17*, 49, 1990) pointed out that a plasma-sheet "bubble", a flux tube that has lower $PV^{5/3}$ than its neighbors, will move earthward, toward the direction of smaller flux tube volume.
  - Analogous to the upward motion of a bubble in a liquid.



- A blob, which is a flux tube with $PV^{5/3}$ higher than its neighbors, moves out from Earth.
- *Chen and Wolf* (*JGR*, *98*, 21409, 1993) suggested that bursty bulk flows were bubbles.
- *Chen and Wolf* (*JGR*, *104*, 14613, 1999) developed a theory of bubble dynamics, visualizing a bubble as a thin ideal-MHD filament.
  - Finite conductance earthward boundary.

Motion of Thin-Filament Bubble

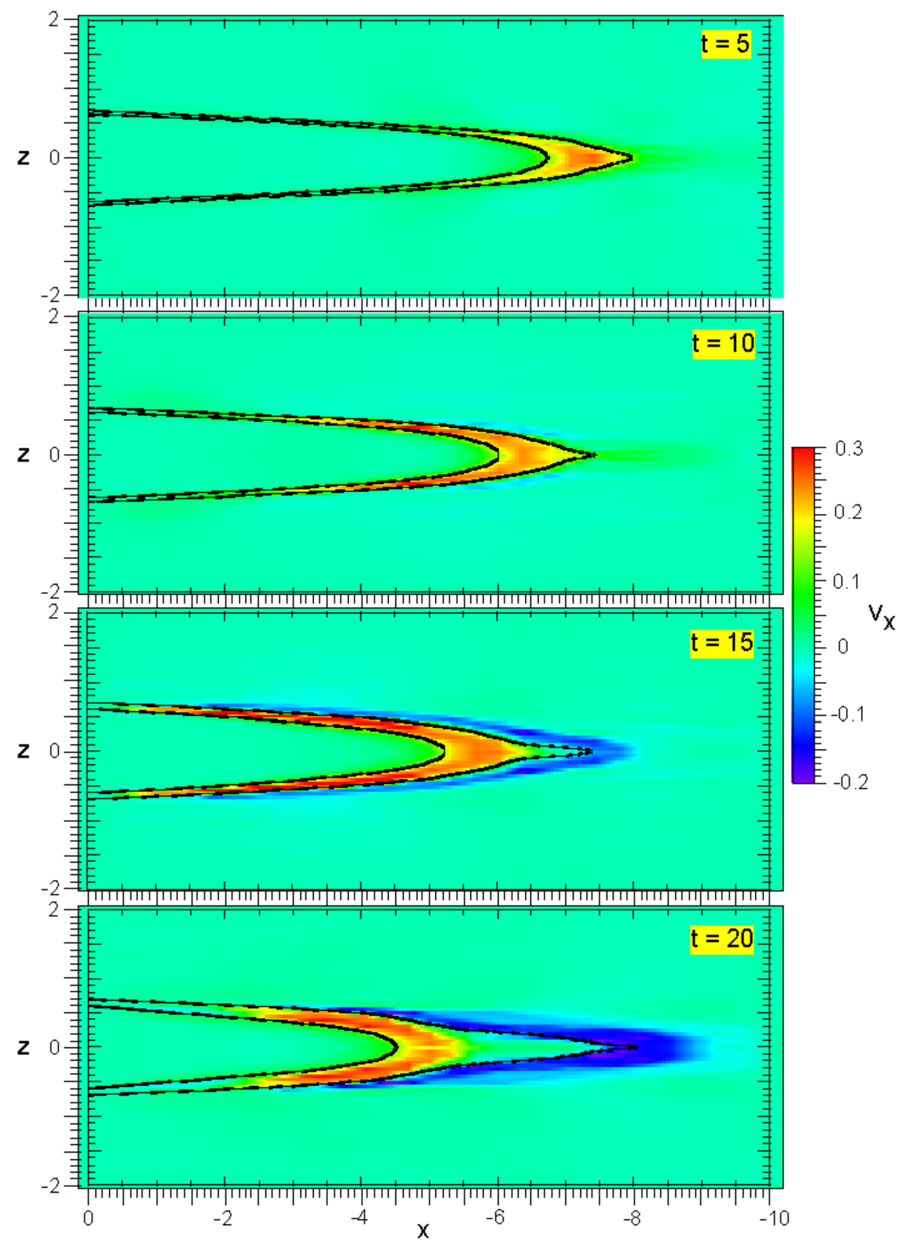
- Field line shortens.
- Second and third panels zoom in on tailward and earthward regions.
- An Alfven wave and a slow mode propagate earthward.
- When the Alfven wave hits the conducting left boundary (ionosphere), that end of filament starts to move equatorward.
- Filament overshoots equilibrium position but eventually settles into equilibrium.
- Simulation was ideal-MHD with some friction between filament and background.



From *Chen and Wolf* (1999).
Numbers represent minutes.

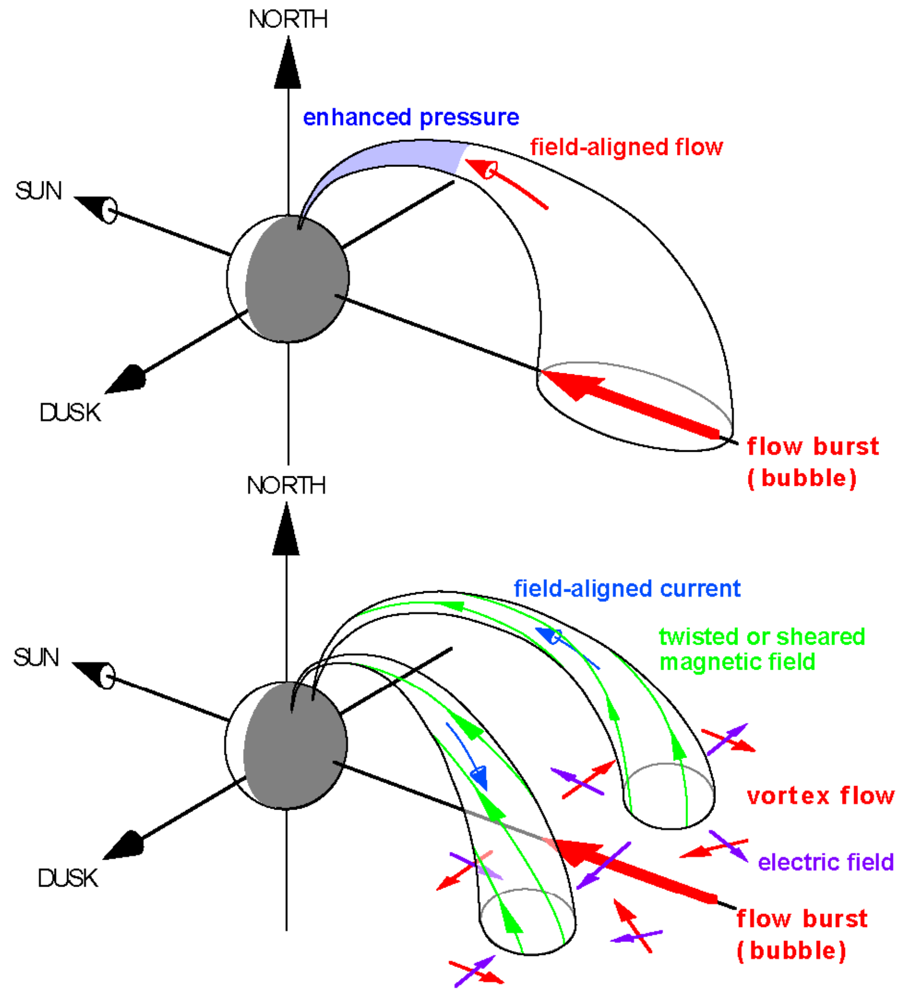
3D MHD Simulation of a Bubble

- *Birn et al*. (*Ann.Geophys*., *22*, 1773, 2004) did a full 3D MHD simulation of a bubble.
- In the plot, colors show sunward velocities V_x.
- Black lines are magnetic field lines.
- These calculations have a perfectly conducting earthward boundary.



Bubble Geometry

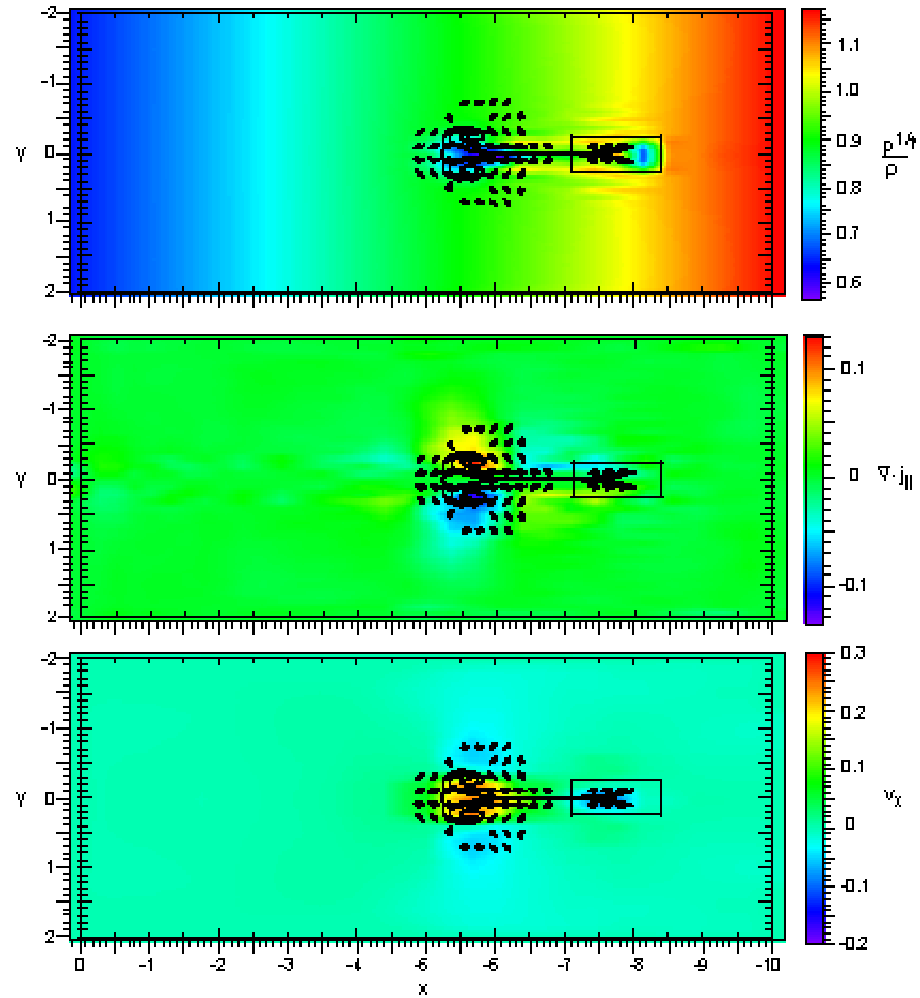
- Flux tubes have to get out of the way of the earthward moving bubble.
  - Creates a twin-vortex flow



From *Birn et al.* (*Ann.Geophys.*, *22*, 1773, 2004)

Shape of Bubble in *xy*-Plane

- Ambient flux tubes ahead of the bubble are deflected to the sides and then fill in the wake.
- Chen-Wolf (1999) thin-filament simulations represent the central part of the bubble.
  – Don't deal with shape
- In 3D MHD calculations, the details of the shape in *xy* plane depend on numerical viscosity.



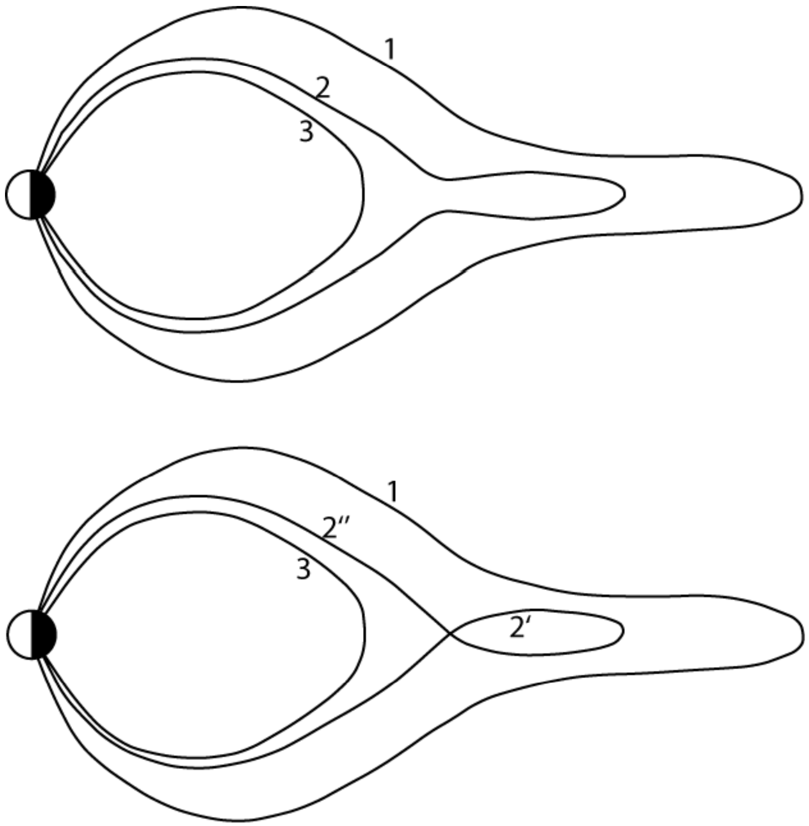
Birn et al. (2004)

Some Bubbles Observed in the Plasma Sheet and in Auroral Streamers

- *Sergeev et al*. (*JGR*, *101*, 10817, 1996) (ISEE)
- *Lyons et al*. (*JGR*, *108* (A12), 2003) (Geotail)
- *Apatenkov et al.* (*Ann. Geophys*., *25*, 801, 2007) (Cluster)
- *Kauristie et al.* (*JGR*, *105*, 10677, 2000) (Wind)
- *Nakamura et al.* (*JGR*, *23*, 553, 2005) (Cluster)
- *Sergeev et al*. (*Ann. Geophys*., *22*, 537, 2004). (DMSP, Polar UVI)

- Most of these are bursty bulk flows, although one is a substorm.

Reconnection as a Source of Bubbles



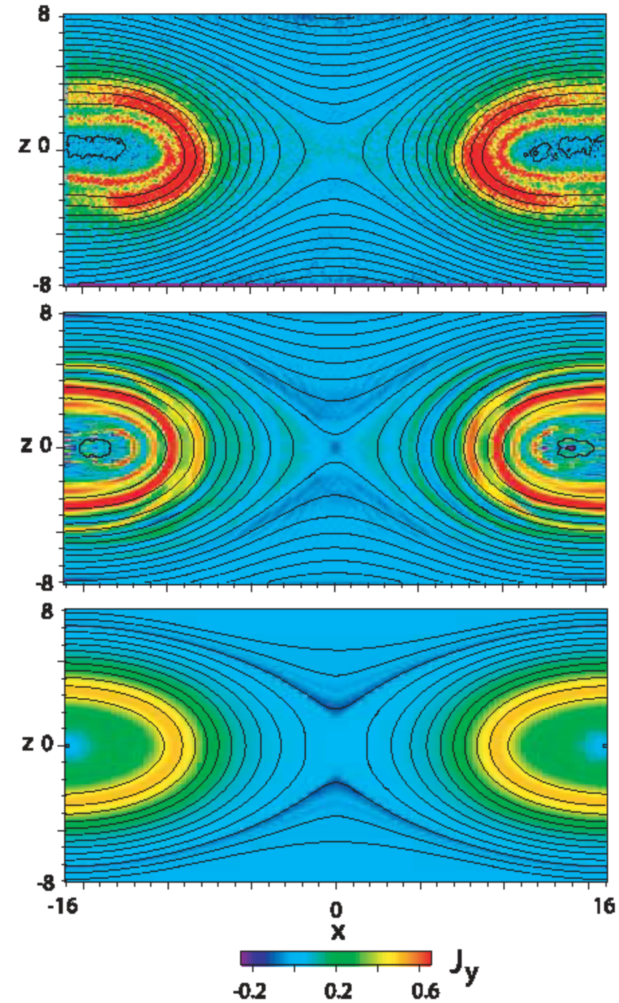
- Suppose reconnection occurs on flux tube 2, splitting it into 2' and 2''.
- In the ideal-MHD approximation, entropy is conserved and

$$P_{2'}^{3/5}V_{2'} + P_{2''}^{3/5}V_{2''} = P_2^{3/5}V_2$$

- But of course reconnection can't occur in ideal MHD.
  - Can entropy still be approximately conserved in a more realistic treatment that allows reconnection?

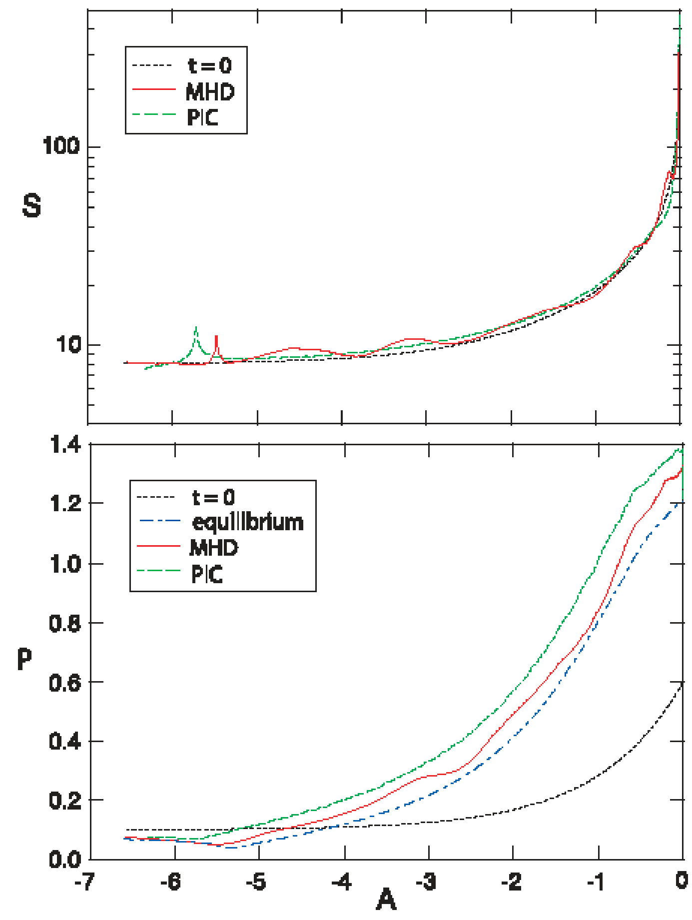
Entropy Conservation in Reconnection

- *Birn et al*. (*Phys. Plasmas*, *13*, 092117, 2006) looked at entropy conservation in a simulation of the "Newton Challenge", which represented reconnection in a Harris-sheet idealized situation.
- The plot shows final state in PIC simulation (top), MHD simulation (middle), and minimum energy configuration for the same entropy function.



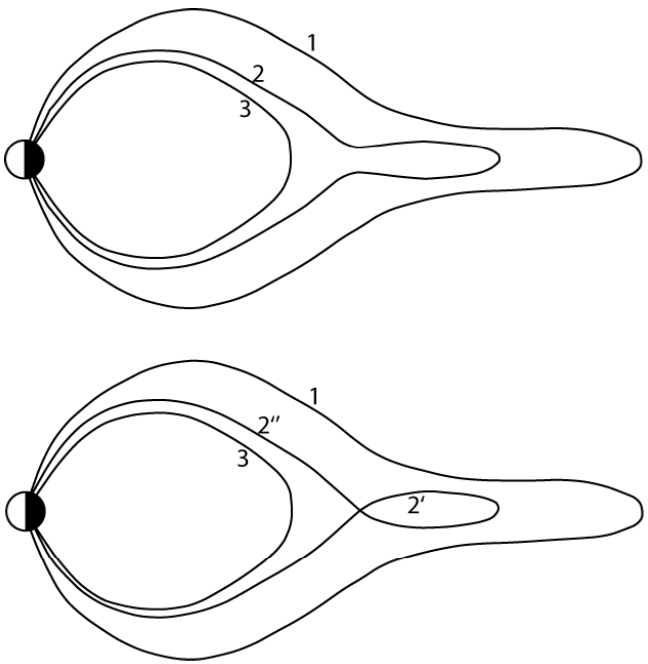
Entropy Conservation in Reconnection

- The upper plot looks at $S=P^{3/5}V$ as a function of flux-function *A*, which labels field lines, comparing initial *S* with values after reconnection had occurred.
- PIC and MHD values both differ little from the initial entropies, even though the pressure changed a lot.
- Entropy was approximately conserved on nearly all flux tubes.
- Birn showed similar simulations at this meeting that included a guide field (B_y). Entropy still conserved pretty well.



Birn et al. (*JGR*, *114*, A00D03, 2009)

How Does Closed-Field-Line $PV^{5/3}$ Change in Reconnection?



- Consider thin flux tube centered on line 2.
- The conclusion from Birn's work is that we should expect

$$P_{2'}^{3/5}V_{2'} + P_{2''}^{3/5}V_{2''} \approx P_2^{3/5}V_2$$

so that

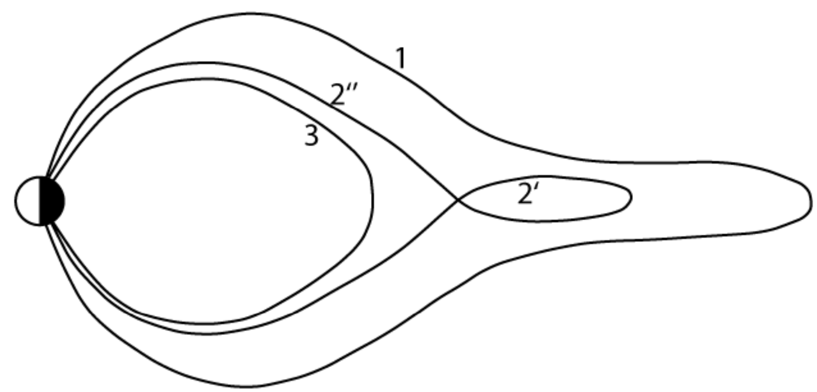
$$P_{2''}V_{2''}^{5/3} < P_2V_2^{5/3}$$

which means that the new closed flux tube (2'') has lower entropy than the old one (2).
- Therefore, reconnection creates a bubble.
- Standard reconnection theory applies: plasma exits the reconnection area at ~ Alfven speed.
- $PV^{5/3}$ is a crucial boundary condition for RCM.

- Identification of the outflow as a bubble helps you visualize what happens to the ejecta in its interaction with the high-flow region near the Earth.
- For mathematical model of a reconnection process that produces a bubble, see
  - *Sitnov et al*. (*GRL 32*, L16103, 2005) (2D equilibrium solution)
  - *Sitnov et al*. (*GRL*, *34*, L15101, 2007) (kinetic theory solution)

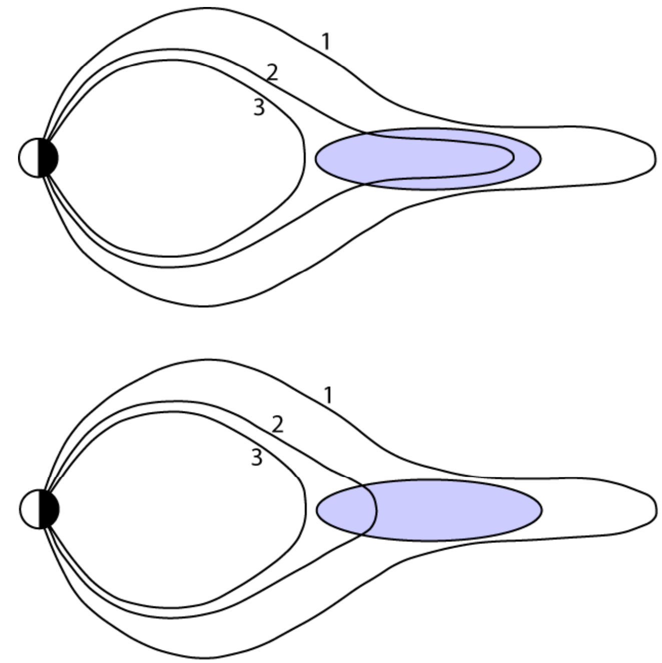
Effect of a Guide Field on the Plasmoid

- A substantial B_y can make what looks like a plasmoid in 2D into a closed-field-line structure that crosses a section of the plasma sheet as a flux rope.
- This structure has a very large flux tube volume, which makes it a "blob" of high entropy $PV^{5/3}$.
  – It will move antisunward, which is what a plasmoid would do.



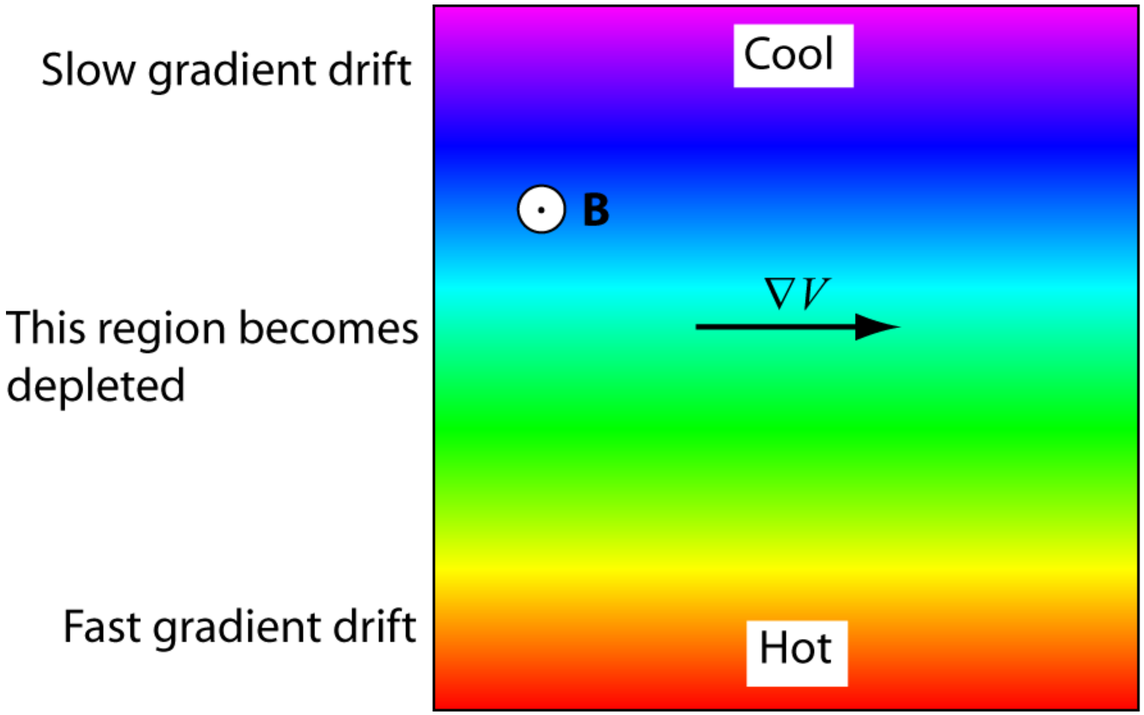
Could Tail-Current Disruption Create a Bubble?

- Suppose tail-current disruption (*Lui et al*, *JGR*, *97*, 1461, 1992) happens in the shaded area, and a positive E_y appears there, the result of anomalous resistivity (or whatever) in the region of positive J_y.
- Now consider flux tube bounded by 1 and 2, and another bounded by 2 and 3.
- Then field line 2 will ExB drift earthward, and equatorial B_z will increase between lines 2 and 3.
- The volume of the tube bounded by 2 and 3 will decrease, creating an earthward-moving low-entropy bubble.
- The volume of the tube bounded by 1 and 2 will increase, creating a tailward-moving high-entropy blob.



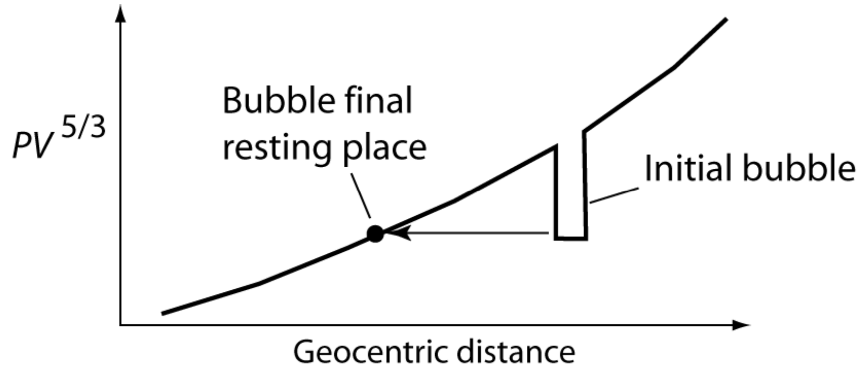
- Between the earthward-moving bubble and the tailward-moving blob, the plasma sheet should thin, possibly creating an X-line.

Lyons Mechanism for Reducing $PV^{5/3}$



7. Restrictions on Bubbles That can Reach the Inner Magnetosphere

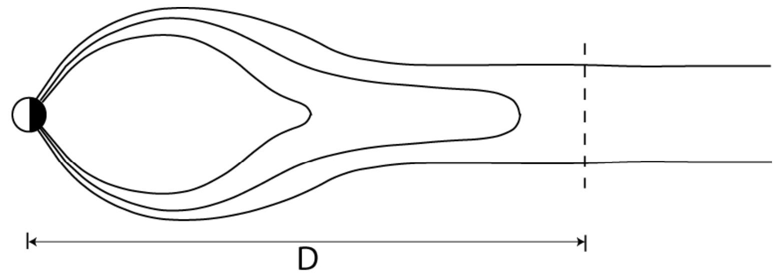
A Bubble's Final Resting Place



- If reconnection in the distant tail creates a bubble that has, say, half the $PV^{5/3}$ that the original closed flux tube had, that bubble will move earthward but probably won't penetrate into geosynchronous orbit.
  - If mixing between bubble and background is negligible, the bubble will move earthward only until it finds a location where the background $PV^{5/3}$ matches its own.
    - Transfer between background and bubble reduces the distance of travel.
- This earthward motion of bubbles helps to resolve the pressure balance inconsistency.

Constraints on Injectable Flux Tubes in Reconnection Picture

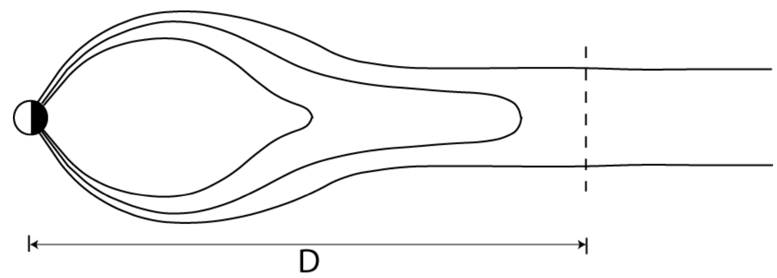
- RCM-E simulations [*Lemon et al.*, 2004] suggest that flux tubes with $PV^{5/3}$>~0.08 nPa$(R_E/nT)^{5/3}$ cannot assume quasi-dipolar form and enter inner magnetosphere.



  - This tentative conclusion needs more analysis and sensitivity study.
- When an X-line occurs at a distance *D* downtail, the *Birn et al*. [2006] result suggests that the entropy on the post-reconnection closed flux tube is equal to the entropy on the part of the pre-reconnection flux tube that lies within *D* of the Earth.
- If reconnection occurs at distance *D* downtail, and entropy is conserved except near the reconnection site, then only a limited number of closed flux tubes near the lobes will have small enough entropy to be capable of transport to the inner magnetosphere.
  - Reconnection of tail-lobe field lines should produce bubbles of very low entropy.

Constraints on Injectable Flux Tubes in Reconnection Picture

- In analyzing storms, one could estimate entropy-related quantities like $\int d\Phi\, PV^{5/3}$ or $\int d\Phi\, P^{3/5}V$ (*)
  for the storm-time ring current, where $d\Phi$ is an element of magnetic flux, given the fact that the similar integral
  $$\frac{3}{2}\int PV d\Phi \qquad (\dagger)$$
  is the particle energy in the ring current.

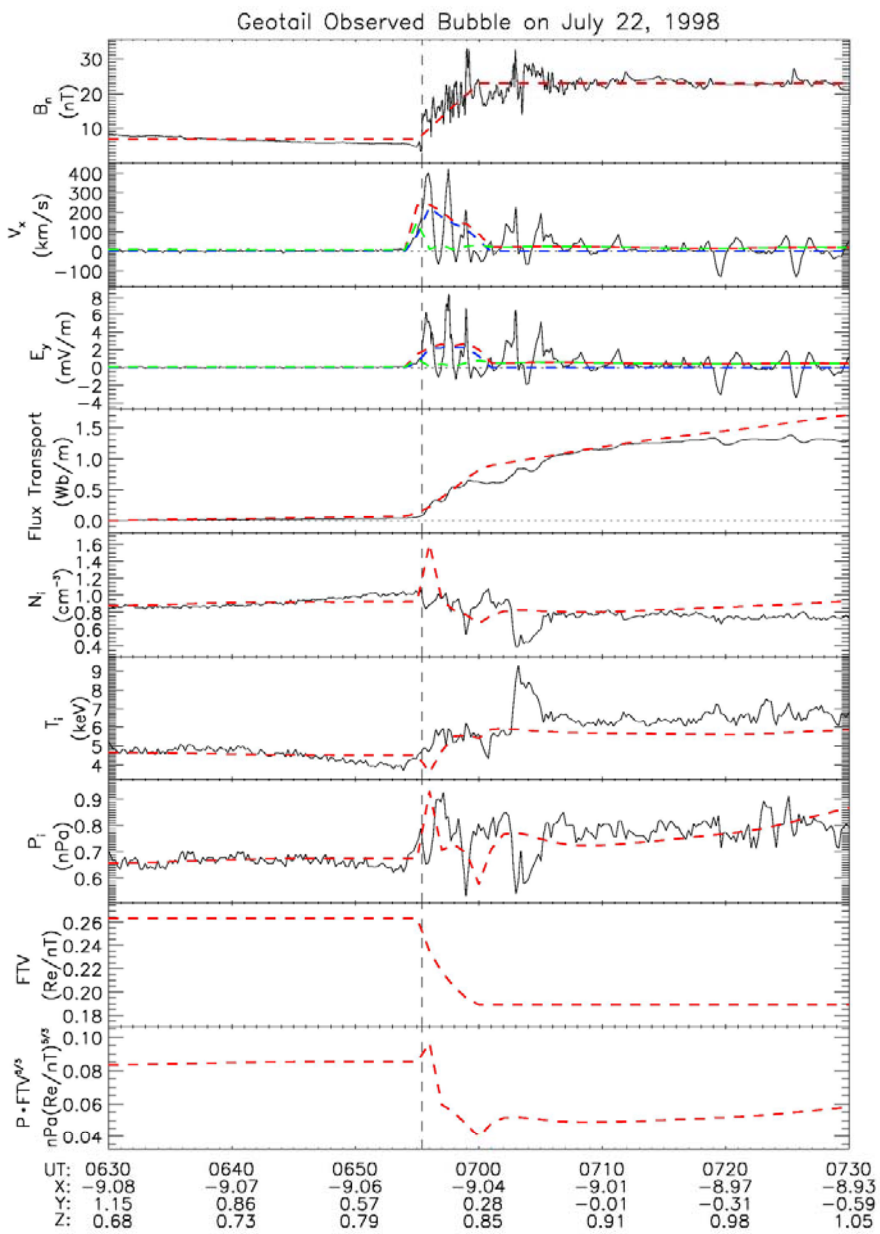


- For a given event, if you can estimate one of the integrals in (*) for an assumed model of how reduced-entropy flux tubes are created in the tail, you can compare it with (†) to see whether the mechanism is strong enough.
- A very rough analysis of this type was carried out by *Kan et al*. (*JGR, 112,* A04216, 2007)

8. RCM Simulation of a Bubble in a Substorm

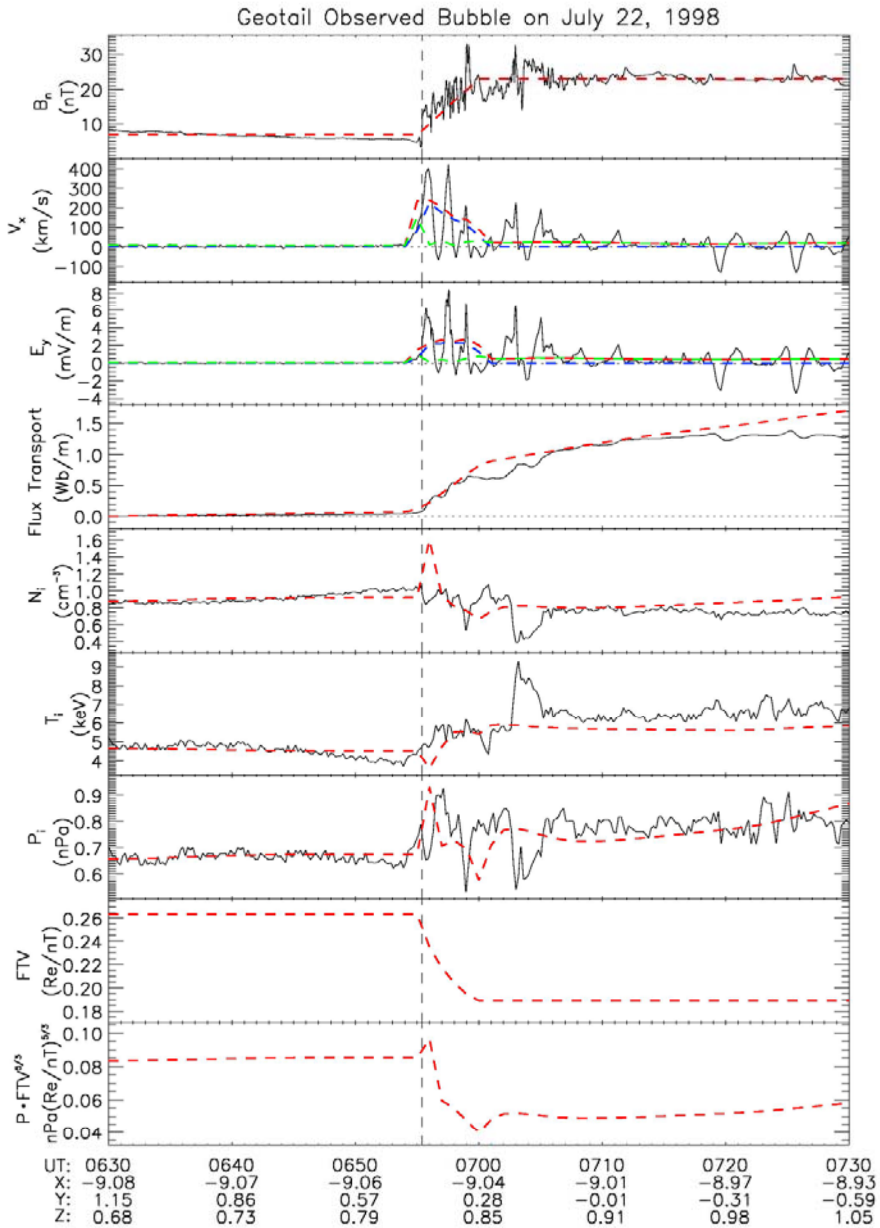
RCM Simulation of 7/22/98 Substorm

- Geotail data from $X \approx -9$, $Y \approx 0$.
- Top panel, which displays B_z, shows extreme stretching, followed by dipolarization starting at about 0655 UT.
- Second panel shows earthward velocity, which becomes basically positive starting about 0654UT.
- Third panel shows E_y calculated from plasma data.
- Fourth panel shows $\int E_y dt$, which is the amount of magnetic flux transported earthward, per unit y.



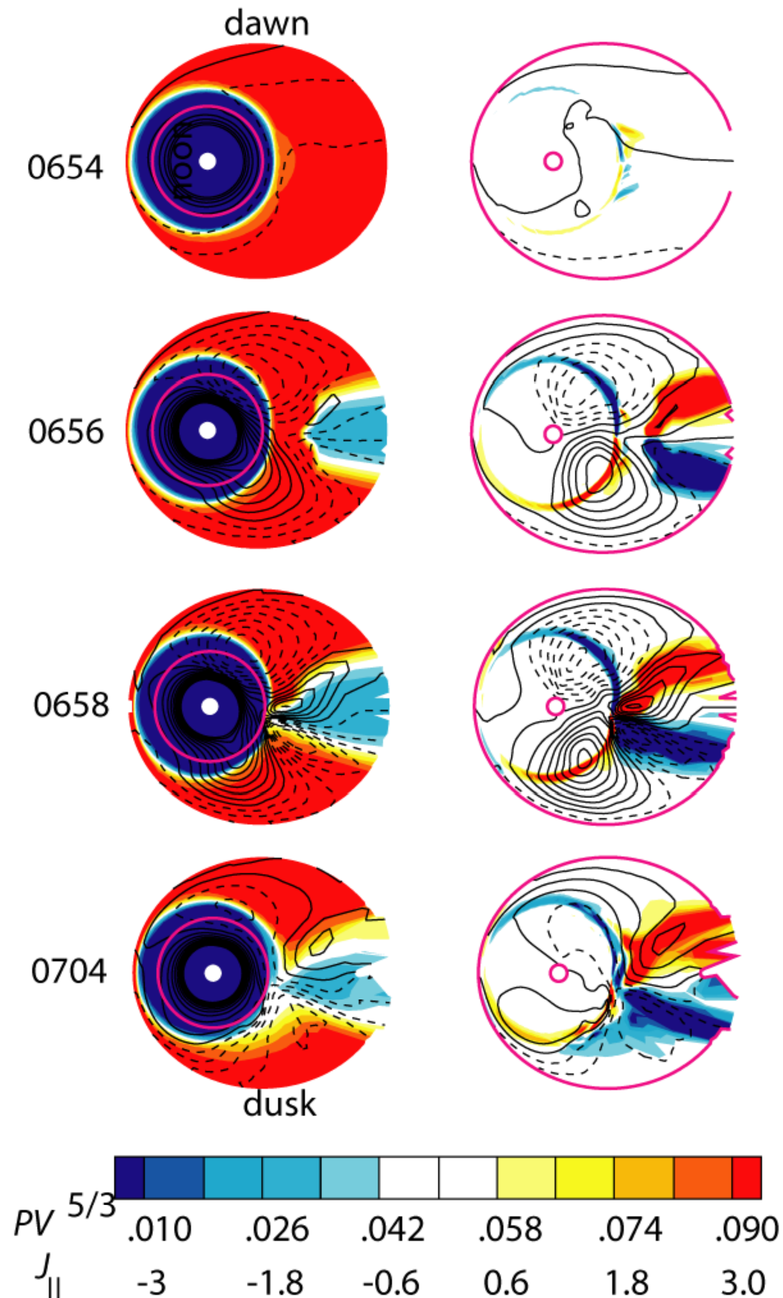
RCM Simulation of 7/22/98 Substorm

- Black curves are Geotail data.
- Dashed red curves are RCM-computed values.
- We adjusted RCM inputs to agree with Geotail, particularly the degree of dipolarization and the correct $\int E_y dt$.
- The last two panels show *V* and $PV^{5/3}$ estimated from the Geotail data, using the method of *Wolf e al*. (*JGR*, *101*, A12218, 2006).
- Note that estimated $PV^{5/3}$ drops significantly in the dipolarization, indicating a bubble.



Sample Simulation Result

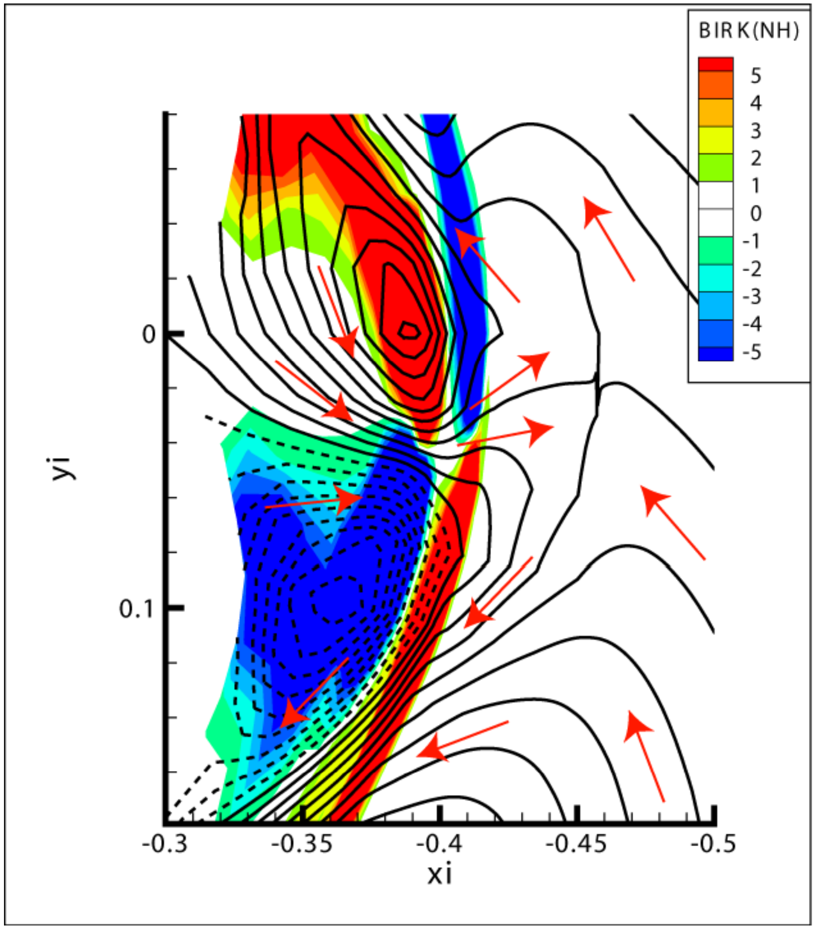
- We have looked at many features of this brief event. An example:
- Dipolarization occurs at t=0655-0700
- Left column shows $PV^{5/3}$ (color) and trajectories for average plasma sheet ion.
  - Blue area is the bubble.
- Right column shows downward Birkeland current and ionospheric equipotentials mapped to equatorial plane.
  - Region-1 sense Birkeland currents flow on the sides of the bubble.
  - Region-2 current flows on plasma sheet inner edge.



(From *Zhang et al.*, *JGR*, in press, 2009)

Another Sample Result

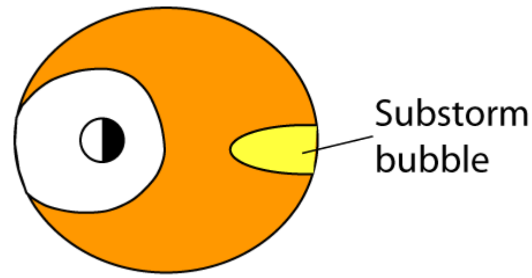
- View of computed ionospheric equipotentials and Birkeland current in the ionosphere. The Sun is to the left and *yi*=0 is local midnight.
- The bubble is moving equatorward (to the right).
- Eastward and westward flow on the eastern and westward sides of the bubble represent plasma getting out of the way of the bubble.



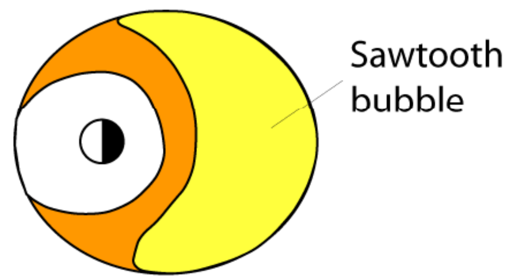
(From *Zhang et al.*, *JGR*, in press, 2009)

9. RCM-E Simulation of a Sawtooth Event

The Difference Between Sawtooth Events and Substorms

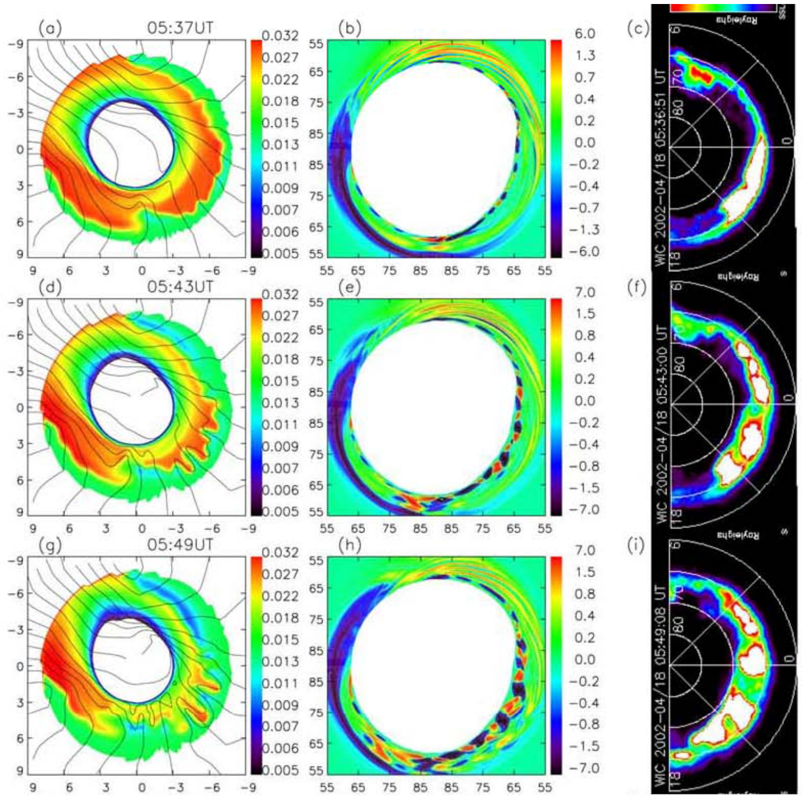


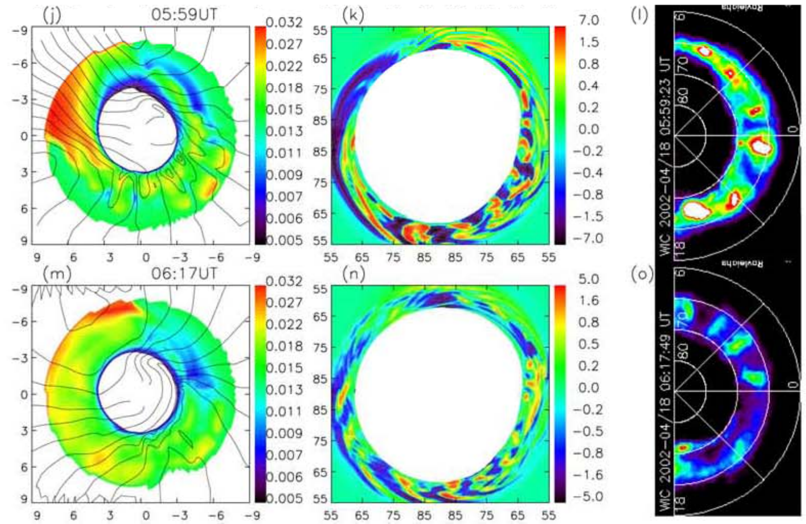
- The early part of a substorm expansion phase covers a narrow range of local time and thus involves a narrow bubble.
- A sawtooth event involves nearly simultaneous dipolarization over a wide range of LT.
  - The dipolarization can only happen if $PV^{5/3}$ decreases.



- The leading edge of any bubble tends to be interchange unstable, since there $PV^{5/3}$ decreases in the direction of increasing V.
- In the substorm case, the bubble moves rapidly earthward, so the interchange instability at the leading edge doesn't have much time to develop.
- In the sawtooth case, interchange instability develops along the leading edge.

RCM-E Simulation and Auroral Images





Left: equatorial $PV^{5/3}$ and potential
Center: Auroral $PV^{5/3}$ and potential
Right: IMAGE FUV/WIC
UT=0537, 0543, 0549, 0559, 0617

- In the 0530 sawtooth, flux tubes of low entropy (green/blue) enter RCM region.
- Interchange instability develops, with outreaching high-entropy fingers (red/orange) and Birkeland currents on the sides of the fingers.
- IMAGE FUV saw series of north-south aligned auroral structures, which we tentatively interpret as resulting from interchange fingers.

Summary

- $PV^{5/3}$ is conserved in the plasma sheet under certain restrictive conditions.
- Steady-state adiabatic-convection solutions exist for the plasma sheet, but their inner regions are much more stressed than statistical models.
- Pressure-balance inconsistency is apparently resolved by processes that break the adiabatic condition in the plasma sheet and by transport of bubbles in from boundary layers.
- Reconnection, current disruption, and differential gradient/curvature drift all break conservation of $PV^{5/3}$ and should be capable of producing bubbles.
- A bubble comes to rest when its $PV^{5/3}$ is the same as its neighbors.
- A bubble entering the inner magnetosphere injects fresh particles and produces characteristic disruptions of the normal E field and current patterns.
- A sawtooth event, which is a very wide bubble, naturally generates interchange fingers.

- The overall conclusion is that bubbles play an important role in plasma sheet transport and dynamics.

Backup Slides

Approaches

- Construct a series of magnetic-field models for the event that are designed to be consistent with measurements during the event.
  - This is the approach used by *Kubyshkina et al*. (*JGR*, *107* (A6), 2002; *JGR*, *113*, A08211, 2008).
  - This is probably the most accurate approach but involves a lot of work.
- We designed a simple little algorithm that estimates V and $PV^{5/3}$ near a measuring spacecraft, using just measurements made by the spacecraft.

Estimation of $PV^{5/3}$ from Single-Spacecraft Measurements

- $PV^{5/3}$ plays a vital role in plasma-sheet transport, but nobody has figured out a way to measure it.
- Average values can be estimated from statistical models of the plasma and magnetic field
  - But that doesn't help in figuring out the changes in $PV^{5/3}$ that occur during an injection event.
- We have made an initial effort at developing a formula for estimating $PV^{5/3}$ locally from measurements on a single spacecraft.
- Start from a simple analytic model of force-balanced tail:

$$A(x,z) = -A_o \cos\left(\frac{\pi z}{2\Delta}\right) e^{-\alpha x} \qquad P(A) = \frac{A^2}{2\mu_o}\left[\left(\frac{\pi}{2\Delta}\right)^2 - \alpha^2\right]$$

$$V(A) = \frac{\pi}{\alpha\sqrt{B_z(A)^2 + 2\mu_o P(A)}}$$

Estimation of *V* and *PV*$^{5/3}$ from Single Spacecraft Measurements

- These 2D analytic expressions, which don't include a representation of the effect of the Earth, don't represent the real inner plasma sheet very well.
- To account for this in a highly approximate way, we replace α by a flexible function of parameters B_z and $(x^2+y^2)^{1/2}$, which can be measured by a spacecraft at the center of the current sheet:

$$V_E(x,y) = \frac{10^C\left(\sqrt{x^2+y^2}\right)^D\left(B_{zE}(x,y)\right)^F}{\sqrt{B_{zE}(x,y)^2+800\pi P_E(x,y)}}$$

Estimation of *V* and *PV*$^{5/3}$ from Single Spacecraft Measurements

$$V_E(x,y) = \frac{10^C\left(\sqrt{x^2+y^2}\right)^D \left(B_{zE}(x,y)\right)^F}{\sqrt{B_{zE}(x,y)^2 + 800\pi P_E(x,y)}}$$

- Algorithms for estimating P_E and B_{zE} from measurements off the equatorial plane:

$$log_{10}\left[P_E(x,y,z)\right] = log_{10}\left[P(x,y,z) + \frac{B_r(x,y,z)^2}{2\mu_0}\right] - G\,\frac{B_r(x,y,z)}{B_z(x,y,z)}$$

$$B_{zE}(x,y,z) = B_z(x,y,z)\sqrt{1 + \frac{B_r(x,y,z)^2}{B_z(x,y,z)^2 + 2\mu_o P(x,y,z)}}$$

  - Fitting to 18 *Tsyganenko* (1996) models for a range of conditions leads to the following choices of adjustable parameters:

$$C = 0.7368, D = 0.7634, F = -0.3059, G = 0.0107$$

for magnetic fields in nT, pressure in nPa, flux tube volume in R_E/nT.

Testing of First-Try Algorithm

- Testing has to be done with models, because nobody has figured out a way to measure flux tube volume.
- T89 models.
  - RMS error in $\log_{10}(V)$~0.09, in $\log_{10}(PV^{5/3})$~0.16.
- Friction code equilibrium model with a depleted channel.
  - Algorithm gave good agreement with this model, which was very different from the Tsyganenko models to which the model was tuned.
- Full MHD thin-filament calculations (*Chen and Wolf*, 1999) that include high-speeds.
  - The formula underlying our estimation algorithm came from assuming equilibrium, with P=constant along magnetic field line, so there is no reason to expect good performance for times of fast flow.
  - Algorithm underestimates the final value of $PV^{5/3}$ by factor ~2 when observed Mach number > 0.2.
- For details, see *Wolf et al*. (*JGR*, 2006).
- We are still working on more tests and improvements.