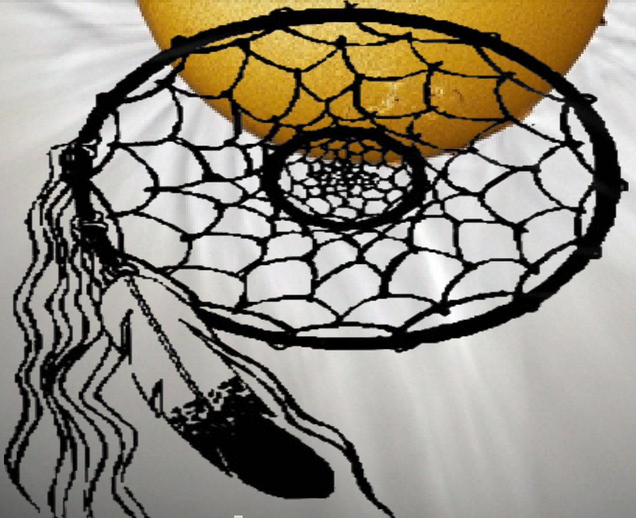
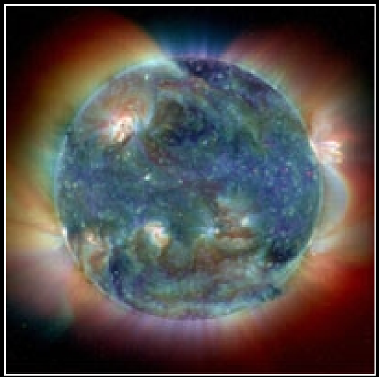
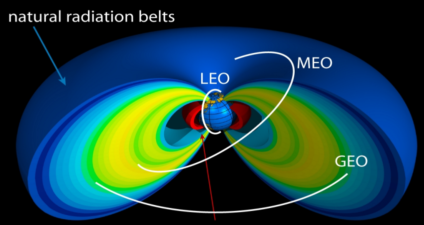


Radiation Belt Data Assimilation: Overview and Challenges

Josef Koller, Humberto Godinez





Los Alamos
NATIONAL LABORATORY
EST. 1943

Outline

Introduction to data assimilation

How does is work?

What is an enKF?

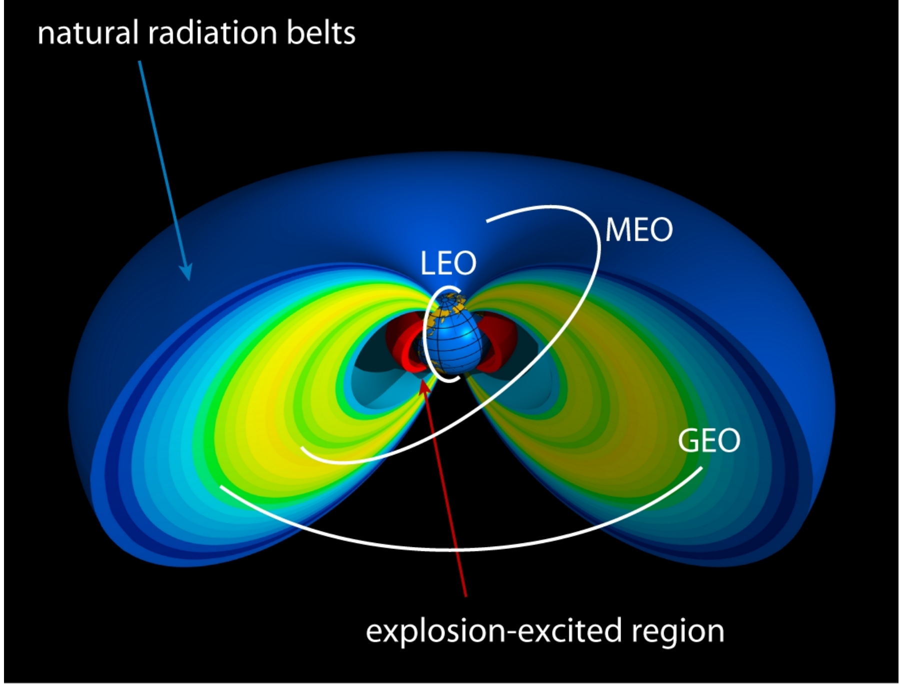
Application to radiation belts and challenges

Radiation belt overview

Combining data with model

Model error and inflation

Summary, Q&A



Data Assimilation in General

Problem set

A physical system (ocean, atmosphere, radiation belt, sun ...)

- Observation of a physical system
- Model of the physical system (an approximate to the time evolution)

We want to increase our knowledge by combining both data and model

model output can be data too!

improved estimate of the (unknown) true state, e.g. radiation belt fluxes

estimate model error and validation

Data assimilation is describing techniques that effectively combine model data in a statistically correct way using their uncertainties

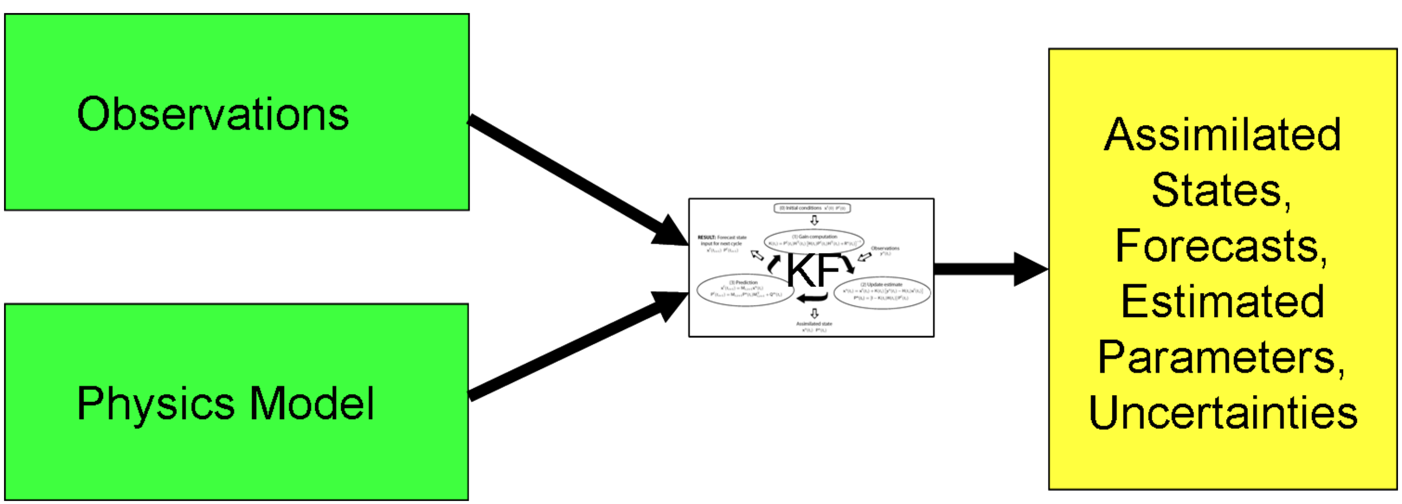
Data Assimilation Basics

Data assimilation is combining data with model using statistical and data analysis tools.

DA includes many different techniques

direct insertion, least square methods, 3D-Var, Kalman Filters and variations.

Main motivation for us: We want to use all information (from models and data) to increase our physical understanding.



Karl Friedrich Gauss

In “Theoria Motus Corporum Coelestium” (1809)

Gauss determined orbits of comets from

- Incomplete astronomical data
- Newtonian mechanics

Gauss invented the “Least Square Method”

Early attempts of weather forecast are based on his method

Key ideas:

- All models and observations are approximate
- Resulting analysis will be approximate as well
- Observations must be optimally combined
- Model is used to preliminary estimate
- Final estimate should fit observation within observation error

Principle of data assimilation

How can we combine data and model in a most effective way?

Maximum likelihood estimate

Bayesian statistics

Least Square method

z1 and *z2* can be information based on observations and/or models.

Note: Final σ is less than either σz1 or σz2 .The uncertainty has been decreased by combining the two pieces of information.

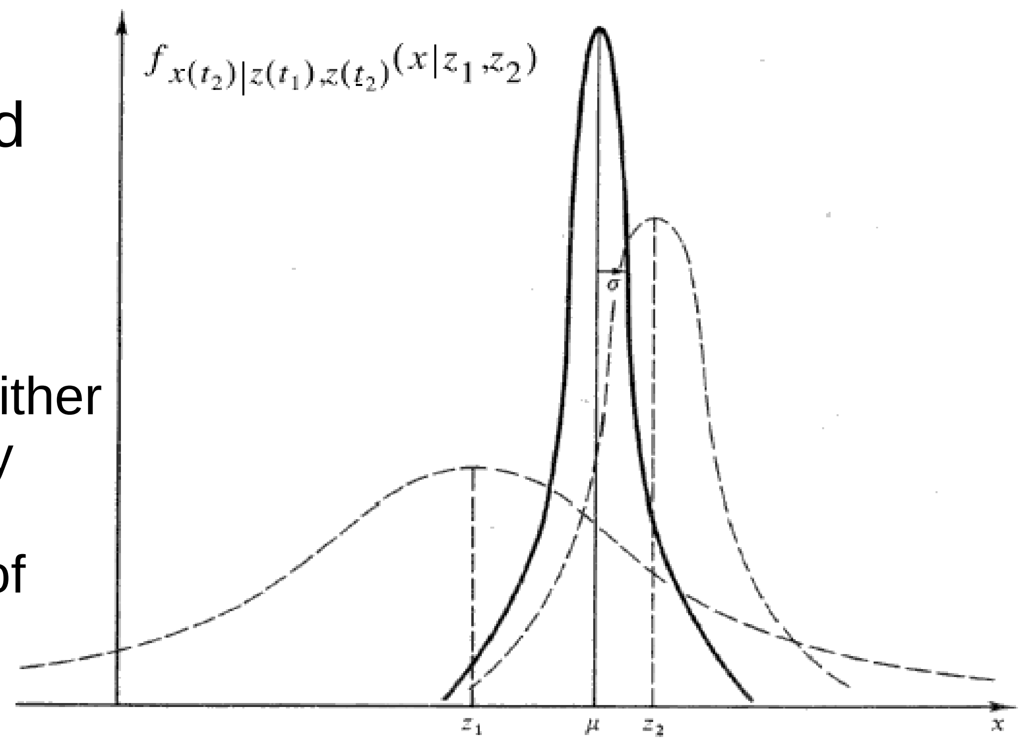
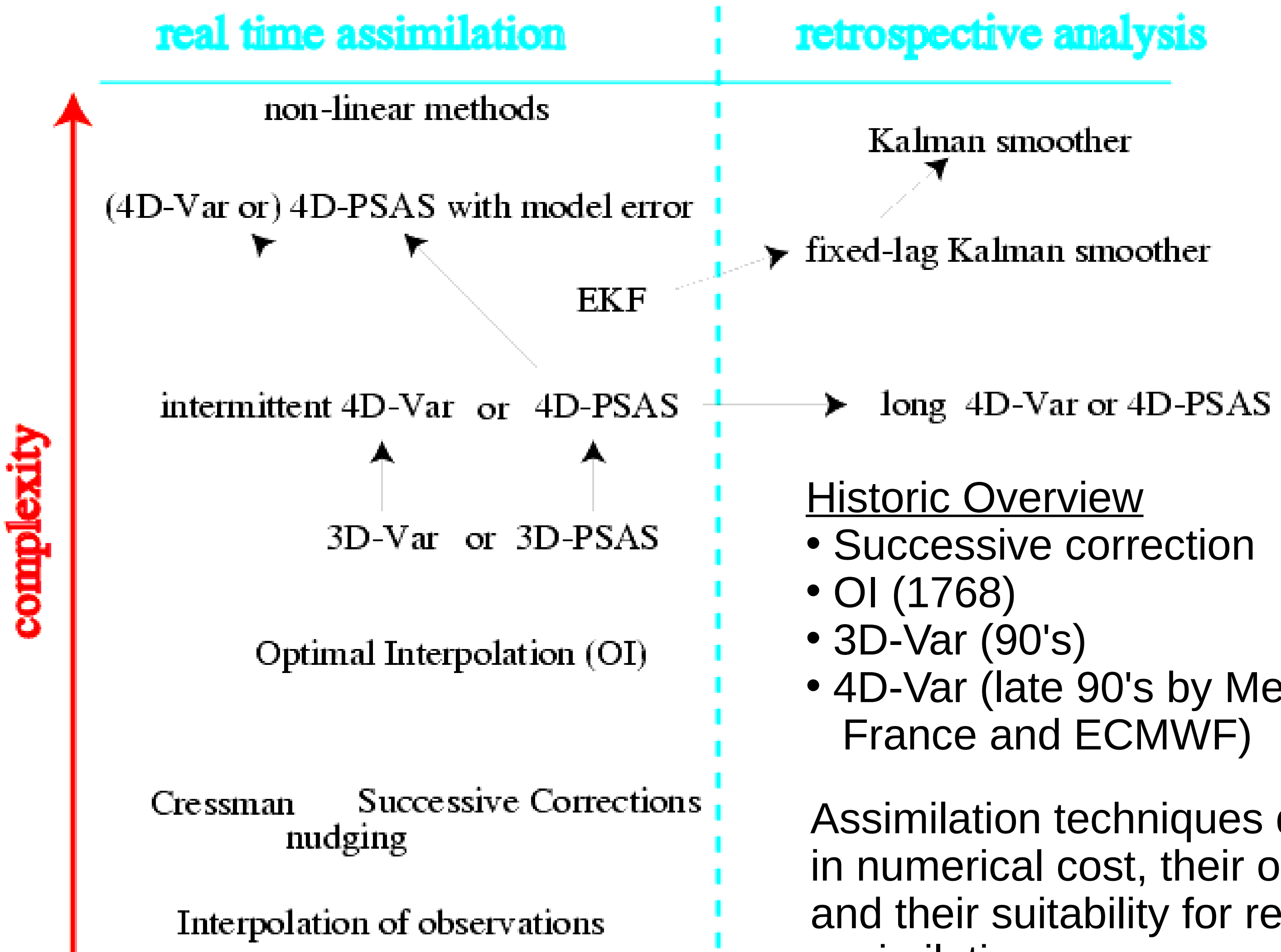


FIG. 1. 6 Conditional density of position based on data z_1 and z_2.

Even poor quality data will provide some information but it will receive only a small weight in the DA algorithm.

Data Assimilation Methods: Algorithms



<u>Historic Overview</u>

- Successive correction
- OI (1768)
- 3D-Var (90's)
- 4D-Var (late 90's by Meteo France and ECMWF)

Assimilation techniques differ in numerical cost, their optimality and their suitability for real-time assimilation.

(Courtesy Bouttier and Courtier 1999).

Linear Kalman Filter Algorithm

(0) Initial estimates

$x^f(0), P^f(0)$

(1) Update estimate $t = t_i$

$$K = P^f H^T [H P^f H^T + R]^{-1}$$
$$x^a = x^f + K[y - H x^f]$$
$$P^a = [I - KH] P^f$$

Assimilated state

$x^a(t_i), P^a(t_i)$

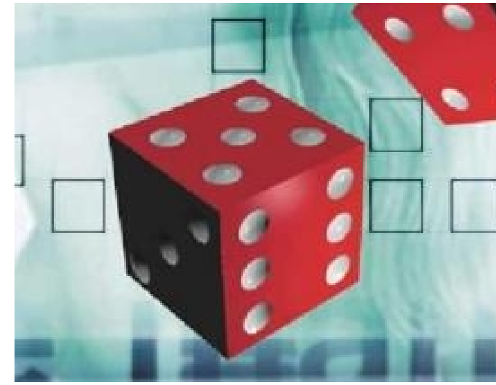
(2) Prediction $t_i \to t_{i+1}$

$$x^f = M_{i,i+1} x^a$$
$$P^f = M_{i,i+1} P^a M_{i,i+1}^T + Q^m$$

Resulting forecast state or input for next cycle

$x^f(t_{i+1}), P^f(t_{i+1})$

Ensemble Kalman Filter (enKF)



enKF is a Monte Carlo method

It describes the covariance matrix by sampling it with ensemble members

Evolves error statistics by ensemble integrations

Computes analysis based on ensemble perturbations and measurement perturbations

Can use any time integration model (here diffusion) as a black box

Converges to Kalman filter with increasing ensemble size

Fully non-linear integration contrary to extended KF

EnKF: Error Covariance Matrix

Define ensemble covariance around the ensemble mean

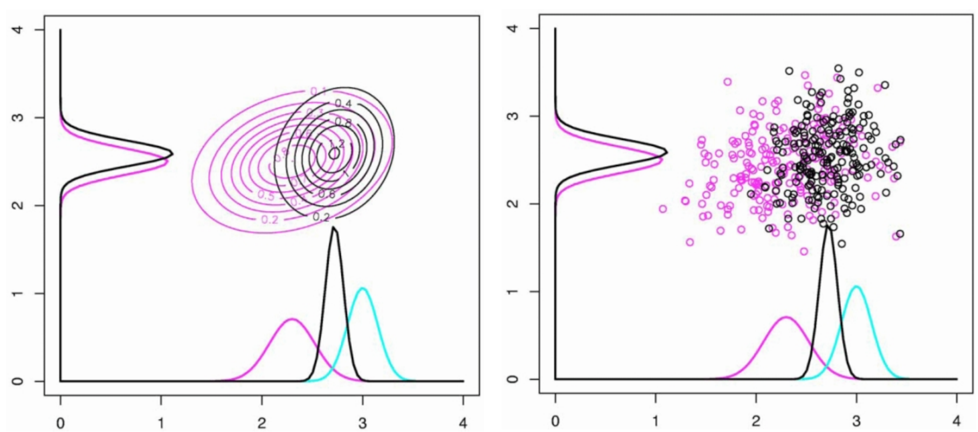
$$\boldsymbol{P}^{\mathrm{f}} \simeq \boldsymbol{P}^{\mathrm{f}}_{\mathrm{e}} = \overline{(\psi^{\mathrm{f}} - \overline{\psi^{\mathrm{f}}})(\psi^{\mathrm{f}} - \overline{\psi^{\mathrm{f}}})^{\mathrm{T}}}$$

$$\boldsymbol{P}^{\mathrm{a}} \simeq \boldsymbol{P}^{\mathrm{a}}_{\mathrm{e}} = \overline{(\psi^{\mathrm{a}} - \overline{\psi^{\mathrm{a}}})(\psi^{\mathrm{a}} - \overline{\psi^{\mathrm{a}}})^{\mathrm{T}}}$$

The ensemble mean $\overline{\psi}$ the best guess

The ensemble spread defines the error variance.

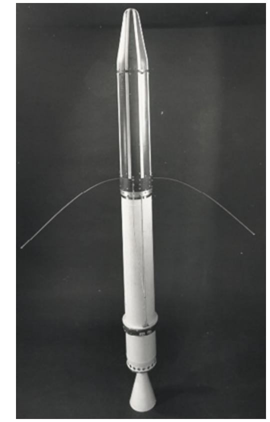
A covariance matrix can be represented by an ensemble of model states (not unique).



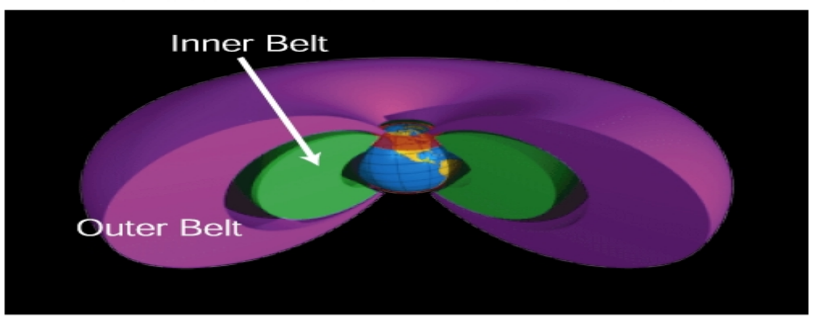
Application: Radiation Belts

Example of applying enKF to radiation belt data and modeling

Discovered accidentally in 1958 by Dr. Van Allen’s cosmic ray experiment onboard explorer I spacecraft.



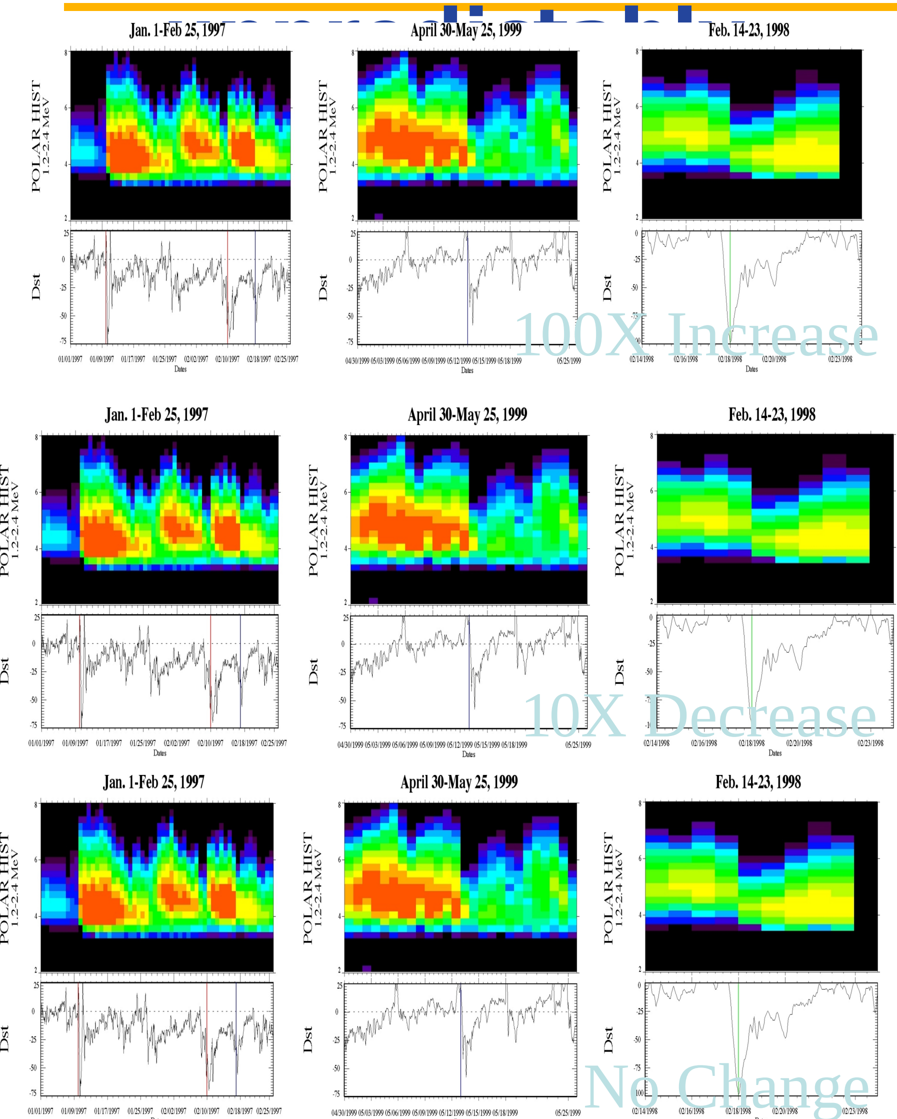
Electron Radiation Belt



B)

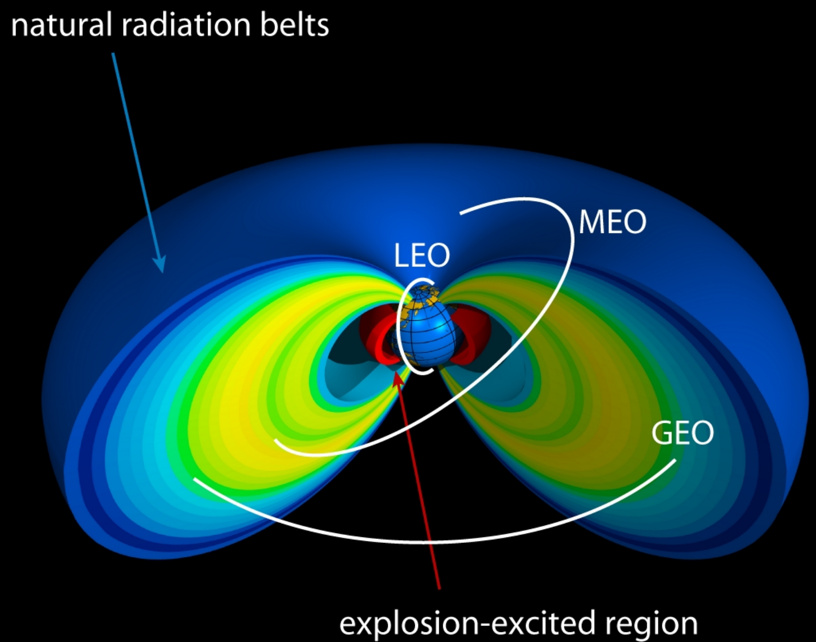
- Energies >0.1 MeV
- Inner belt 1.5-3 Re, Outer belt 3-10
- Slot region: flux minimum near ~3 Re
- Radiation belt electrons = relativistic electrons

Radiation Belt Fluxes change during Geomagnetic Storms - but



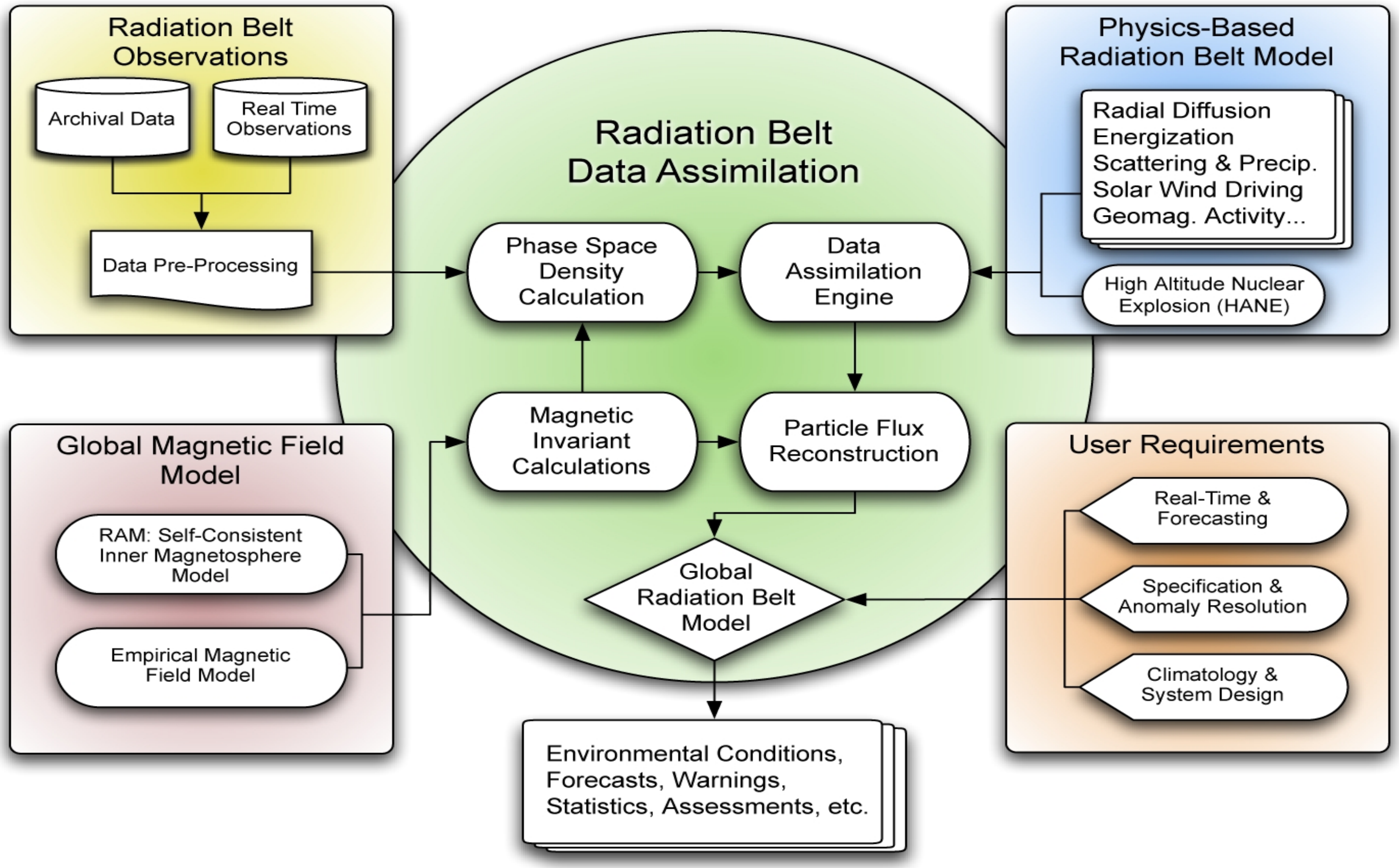
- Geomagnetic storms can increase or decrease radiation belt fluxes or just re-arrange the belts.
- We don’t know why
- Acceleration, transport, and loss mechanisms are not well understood
- Traditional theories have broken down under new observations

DREAM: The Dynamic Radiation Environment Assimilation Model

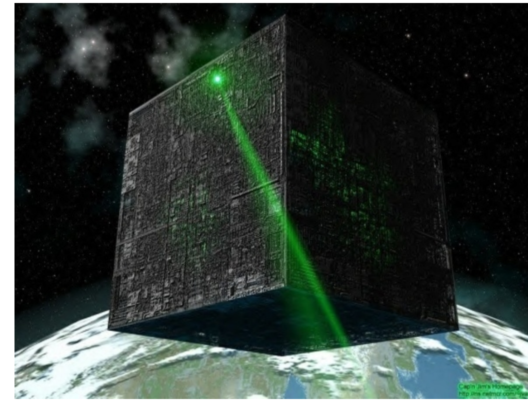


- Developed by LANL to quantify risks from natural and artificial belts
- Uses Data Assimilation with GEO, GPS and other observations
- Couples ring current, magnetic field, and radiation belt models
- Goals: Specification, Prediction, Understanding

DREAM Computational Framework



Our data assimilation framework



Physical model: 1D radial diffusion

$$\frac{\partial \phi}{\partial \tau} = \Lambda^2 \frac{\partial}{\partial \Lambda}\left(\frac{\Delta_{\Lambda\Lambda}}{\Lambda^2} \frac{\partial \phi}{\partial \Lambda} \right) + \Sigma(A, \tau) - \frac{\phi}{\tau}$$

with *DLL* after Brautigam & Albert 2000

$$D_{LL}(Kp, L) = 10^{(0.506\,Kp - 9.325)}\, L^{10}$$

and losses inside the plasmasphere (Carpenter & Anderson 1992)

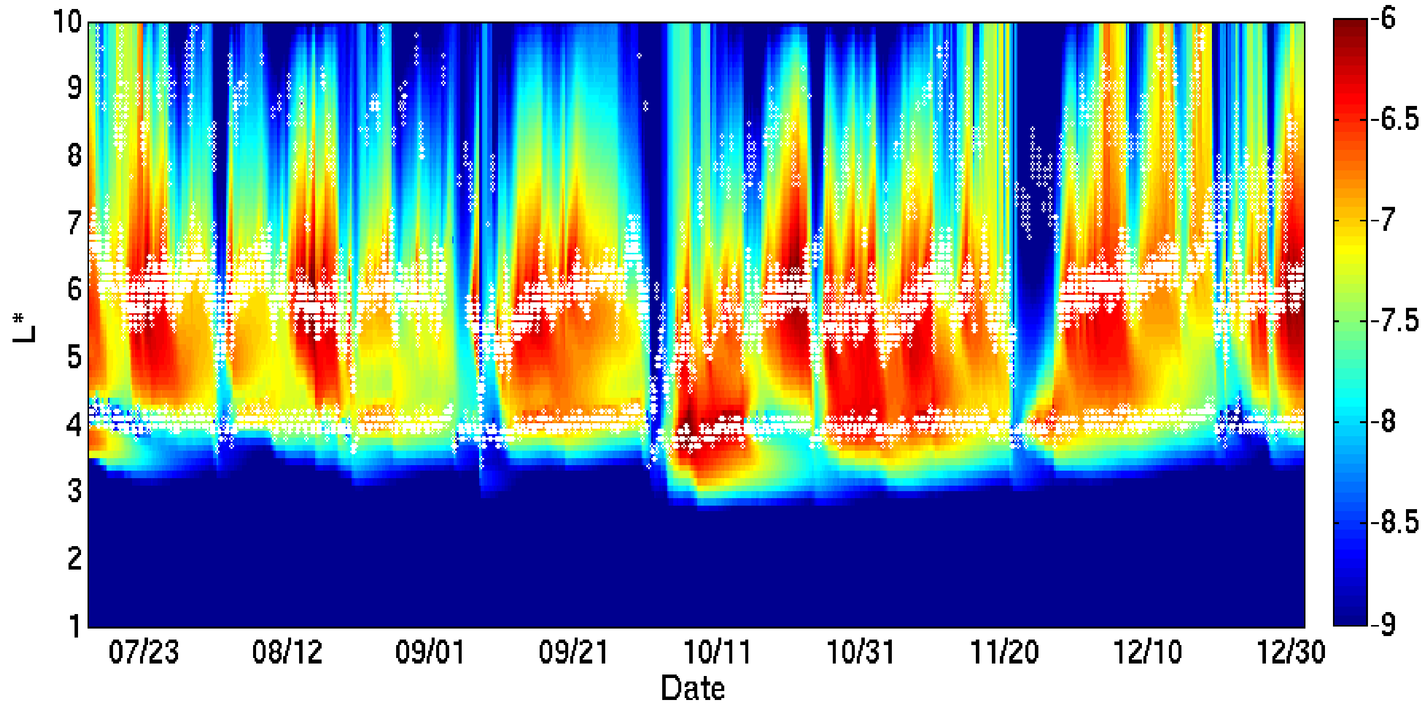
$$L_{pp} = 5.6 - 0.46\,Kp_{\max}$$

Last closed drift shell from T01s model with a strong loss term ~ 10 min

Phase Space Density (PSD) data from 3 LANL Geo, Polar, GPS-ns41

Ensemble Kalman filter with augmented state vector for parameter estimation: time dependent amplitude A of source term

What can data assimilation do?

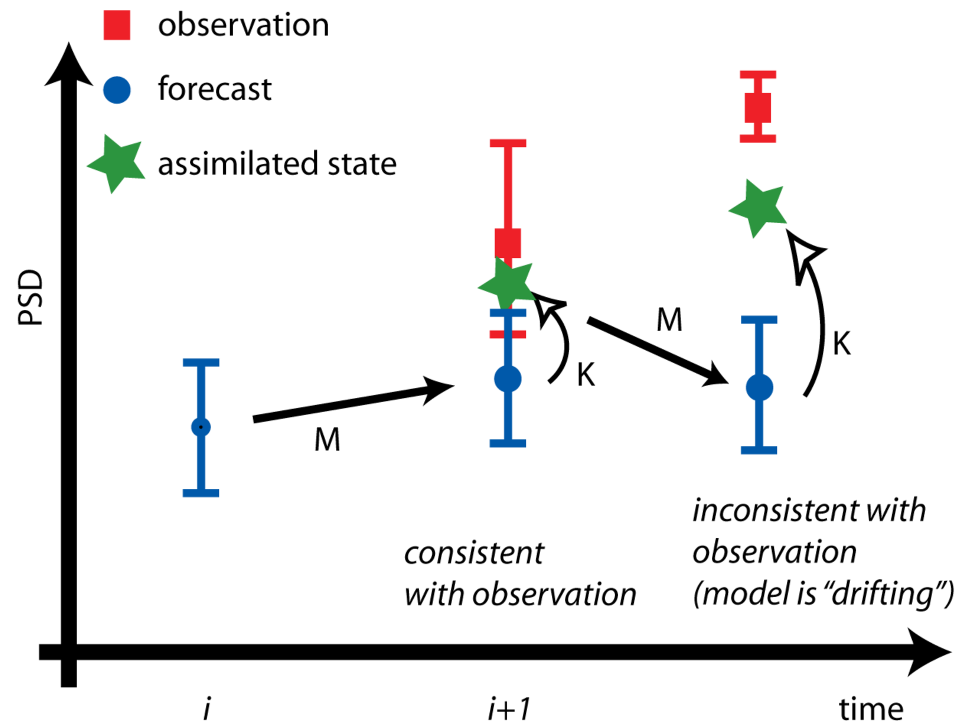


- Estimate the global state as a function of the model and previous observations.
- Fill data voids or holes.
- Predict and forecast future states based on previous observations and a physics based model.
- Estimate model parameters and bias to fit the data.
- Carry along all uncertainties in observations and models.

DREAM Project

Identify Missing Physics in the Model

- Residual Method
  (Koller et al 2007)
- Compare forecast with observations
- Calculate innovation vector *y-Hx (*function of L*, model and data uncertainties)



Residuals can now be used to identify “model drifts”

Is the model forecast consistently too low or too high compared to the observations?

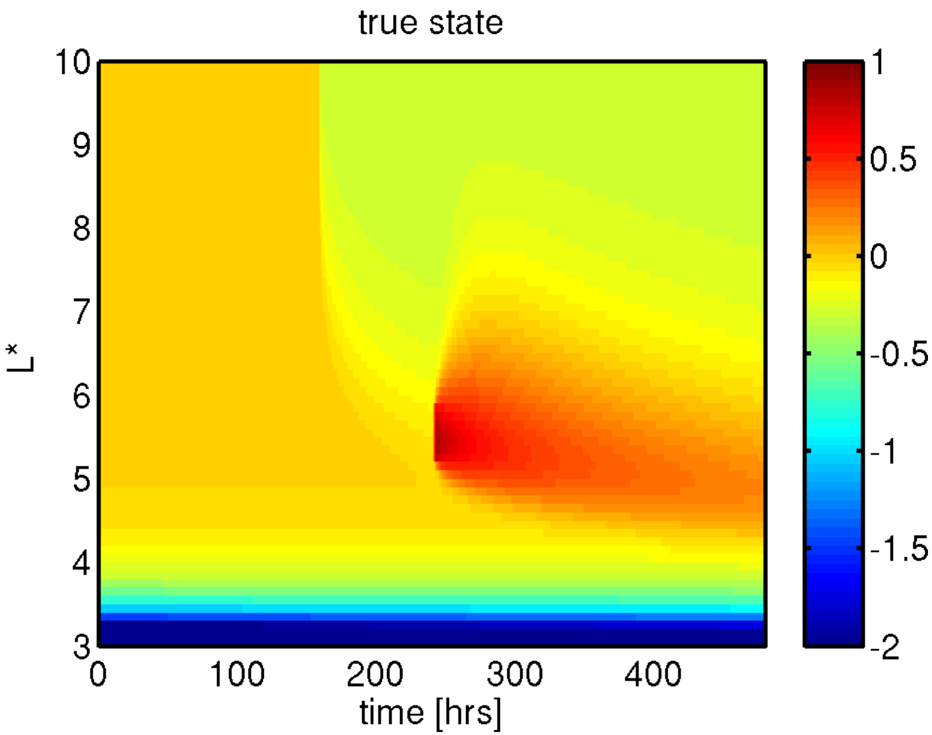
If yes, something most be wrong with the model.

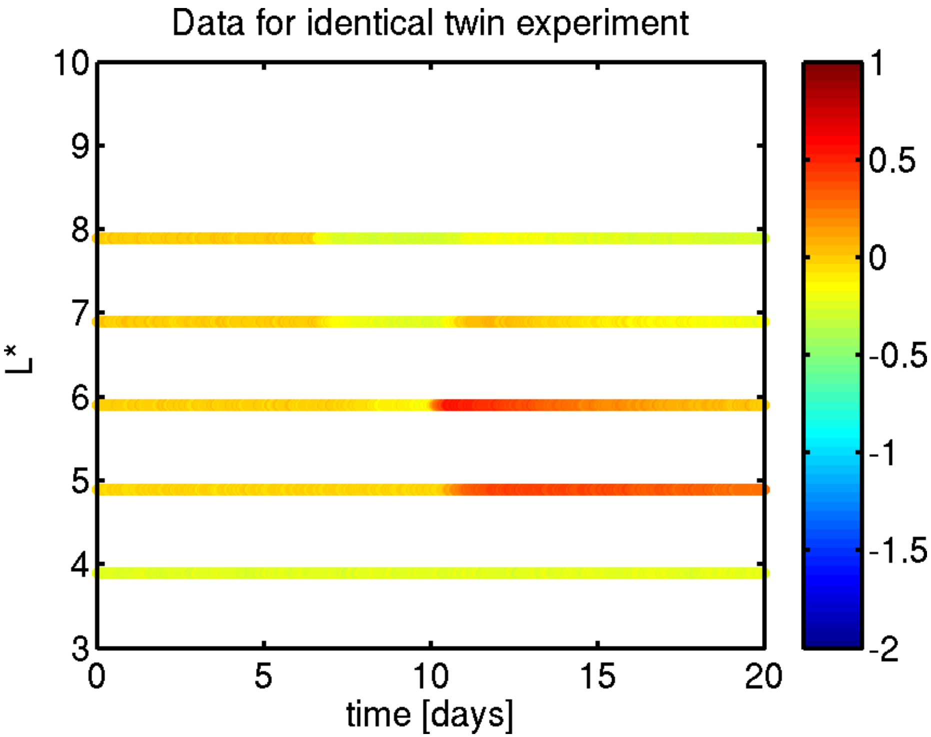
Identical Twin Experiment

Model: Diffusion equation without source:

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(\frac{D_{\mathrm{LL}}}{L^2}\frac{\partial f}{\partial L}\right) \quad \text{with } D_{\mathrm{LL}} = D_0 L^p$$

Reality: with source as shown by measurements





Identical Twin Experiment

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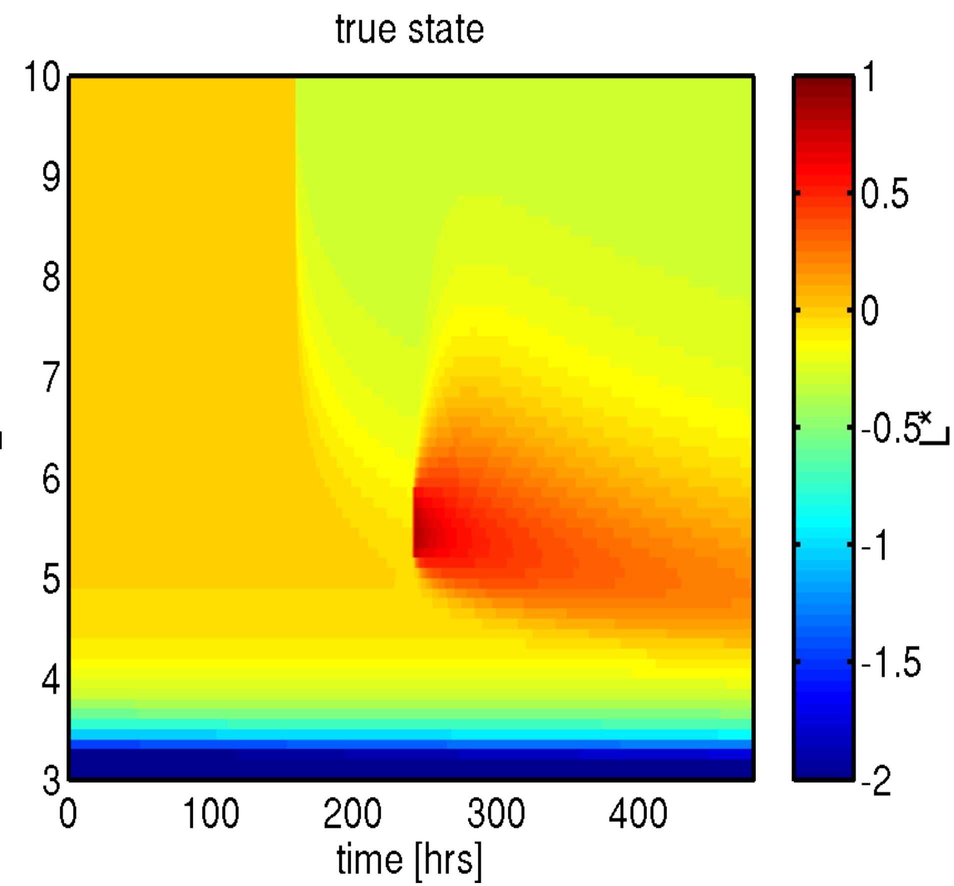
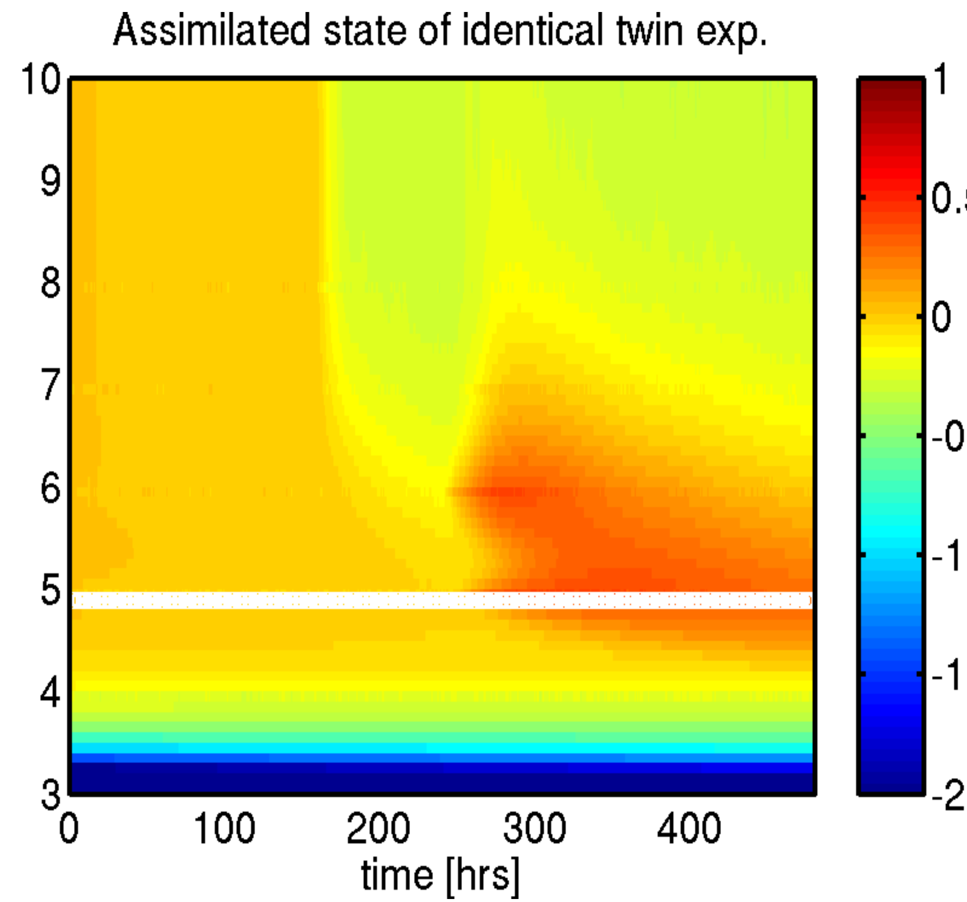
Assimilated state reflects source although process is not in the model

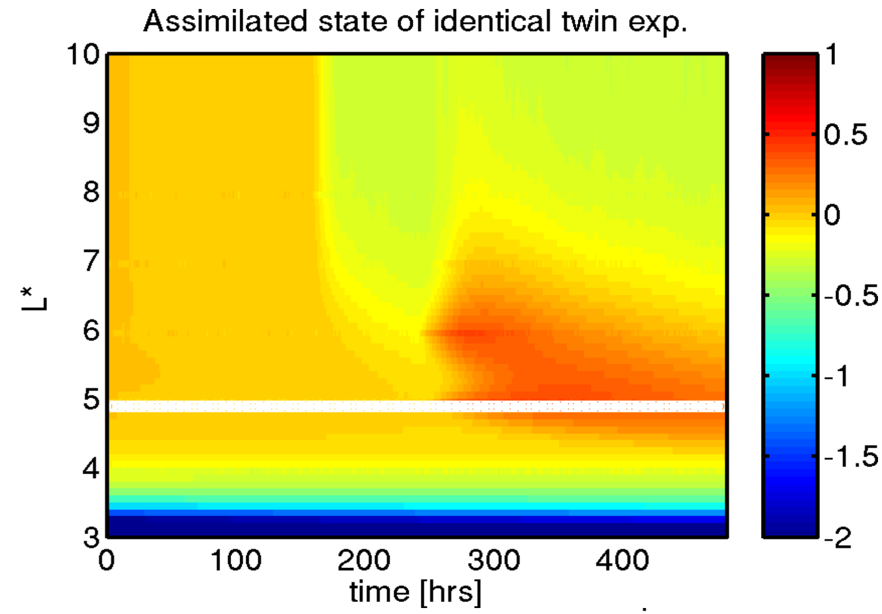


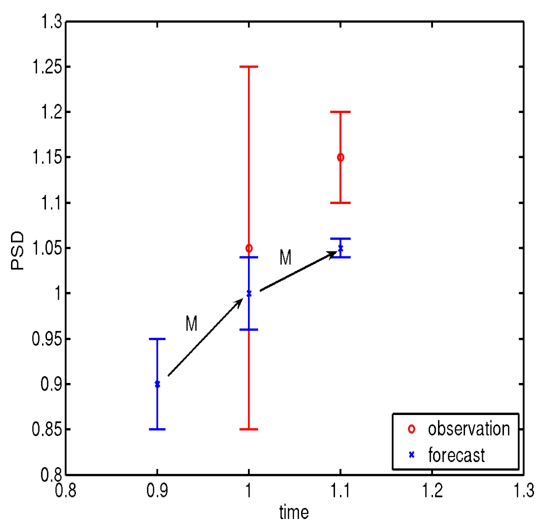


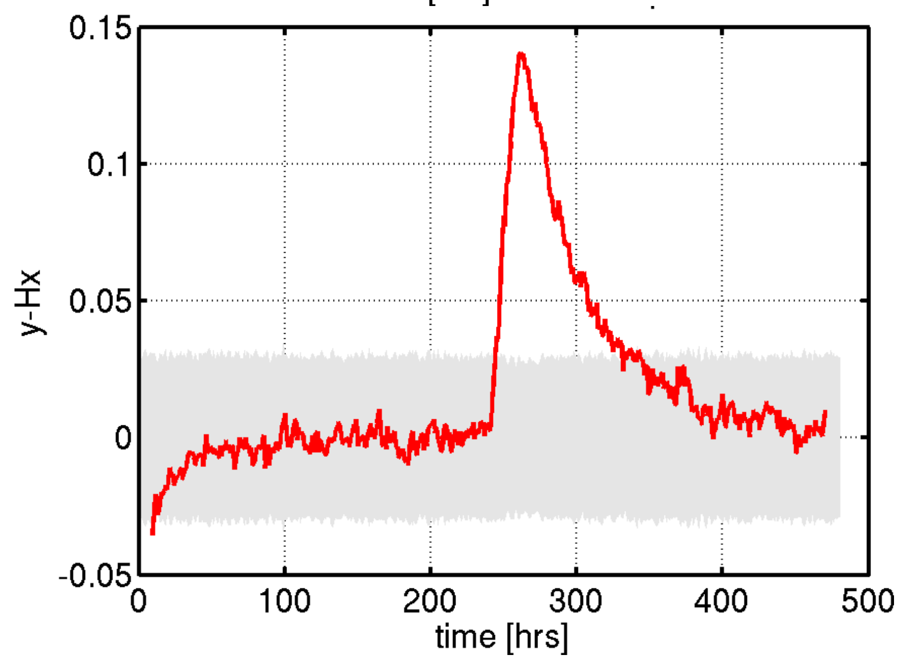
Average Residual tells where model is drifting

Use average residuals of ensemble states to point to a “drifting” physics model where forecasts are inconsistent with data.

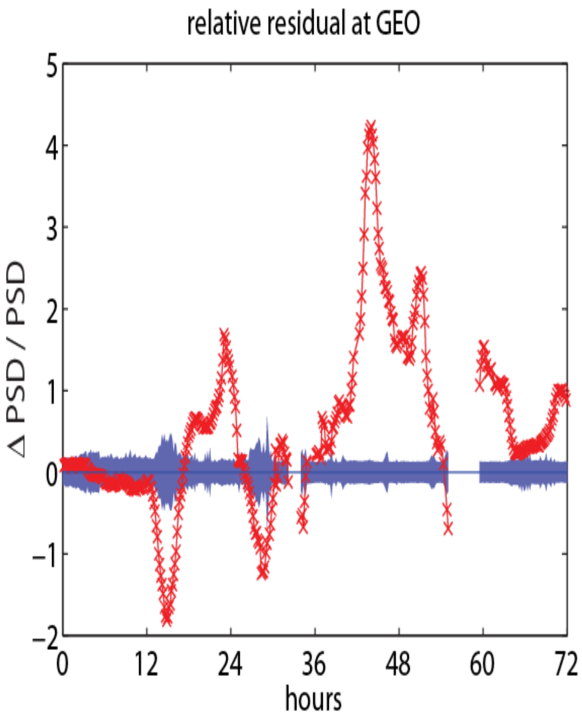
This will help us identify “missing physics” in the model.

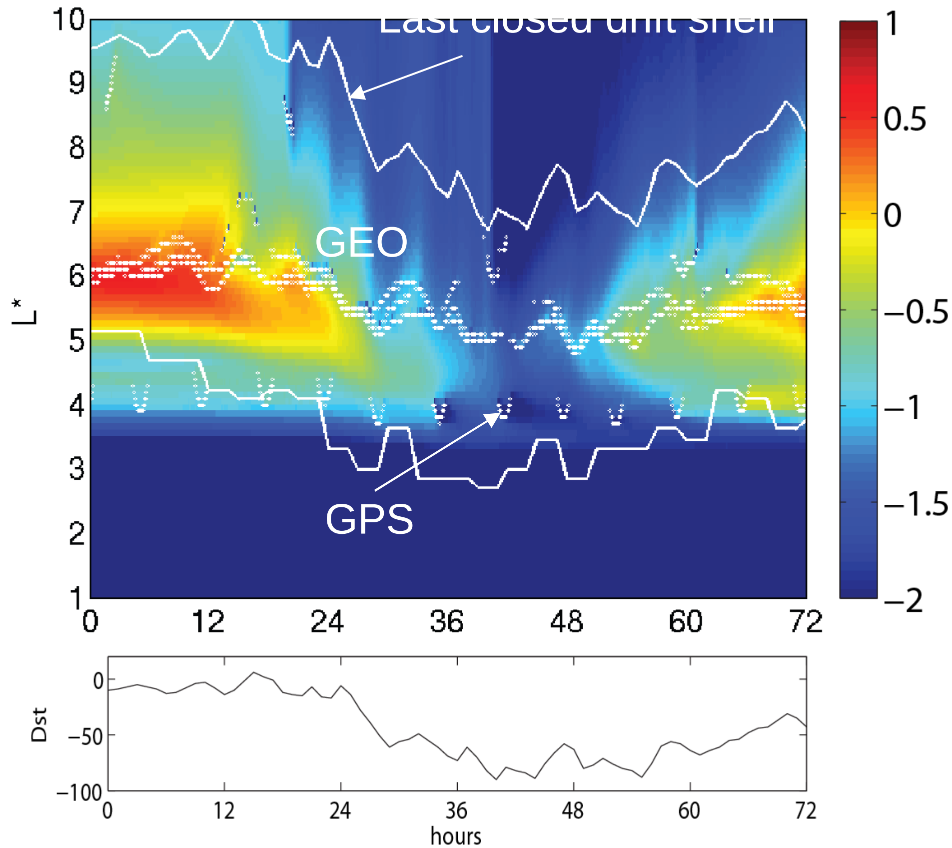






Where are the sources and losses?





Challenges: Accurate Error

accurate data error and model error descriptions necessary

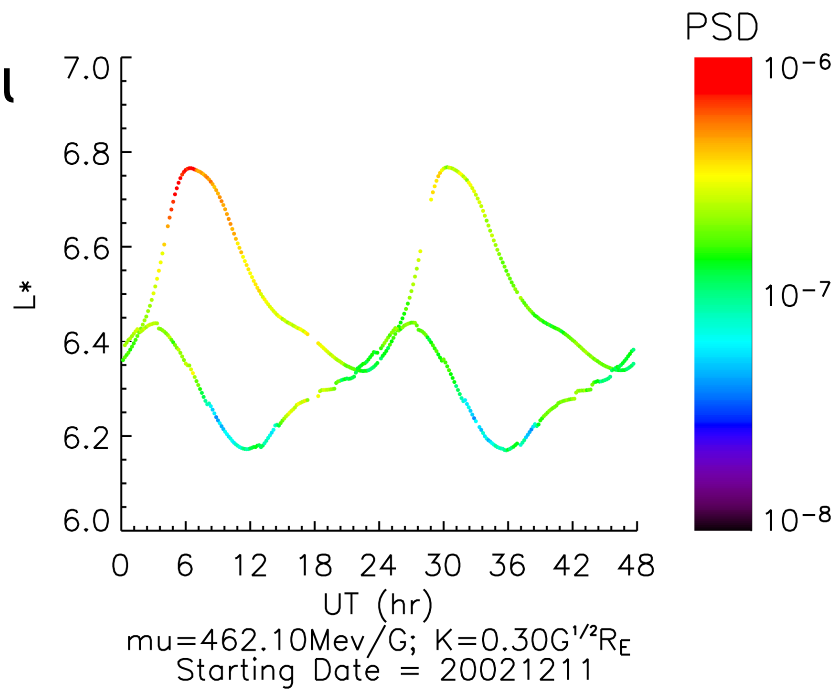
Data error

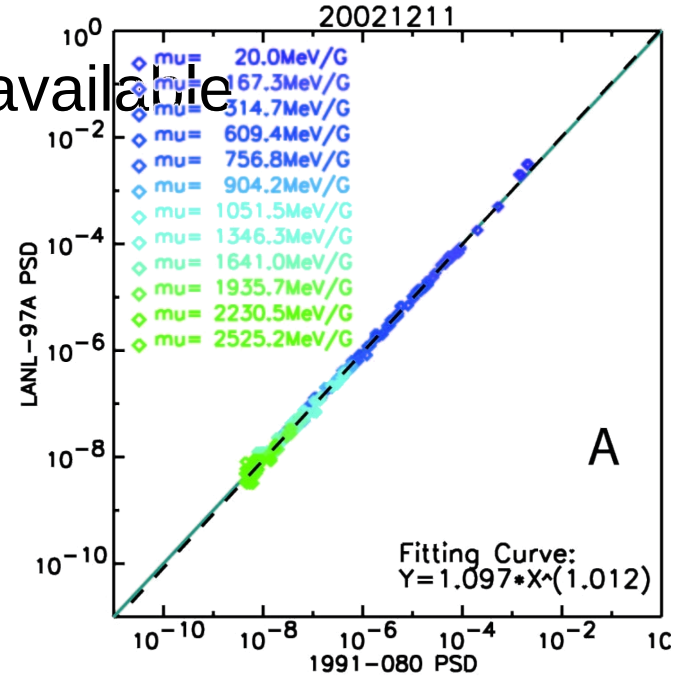
for 1D radial diffusion, can use conjunctions

Model error

can use residual between model forecast and observations

bu available



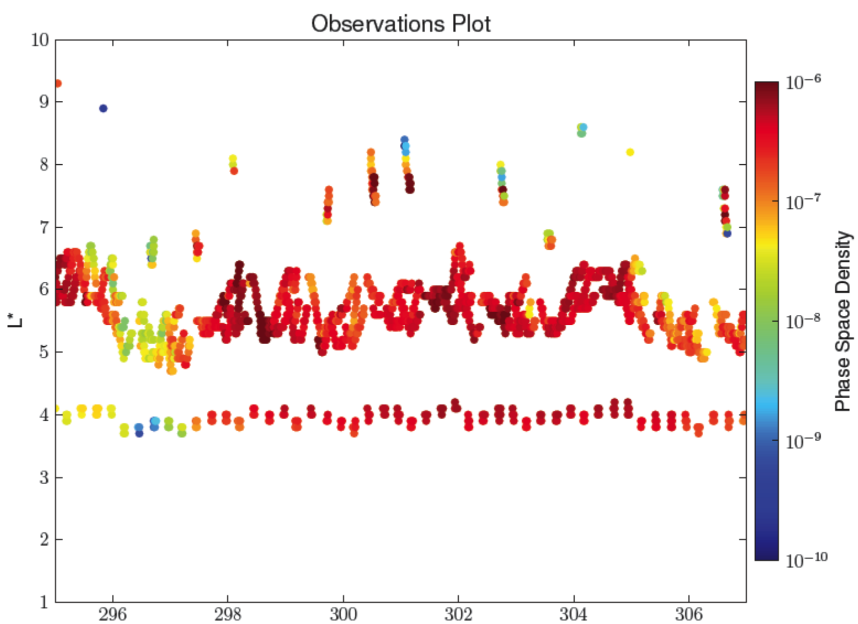


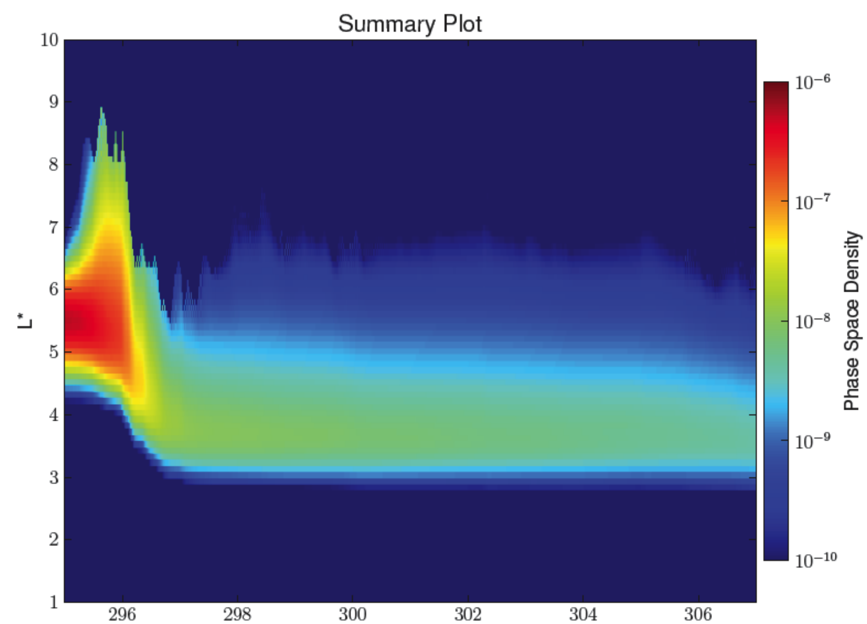
Challenge: Ensemble Inflation

Left: observations only

Right: radial diffusion model only without assimilation

model is clearly inadequate, data assimilation might help





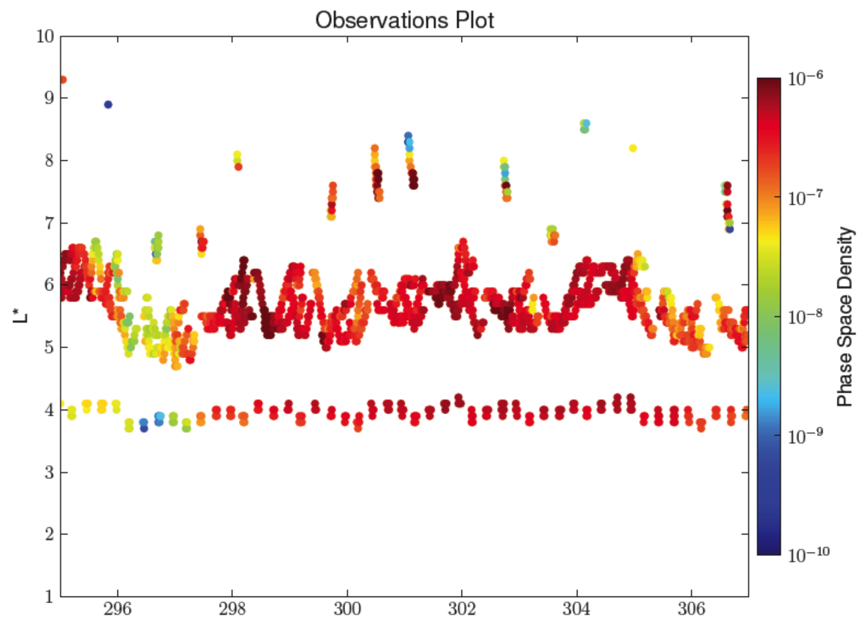
Ensemble Inflation

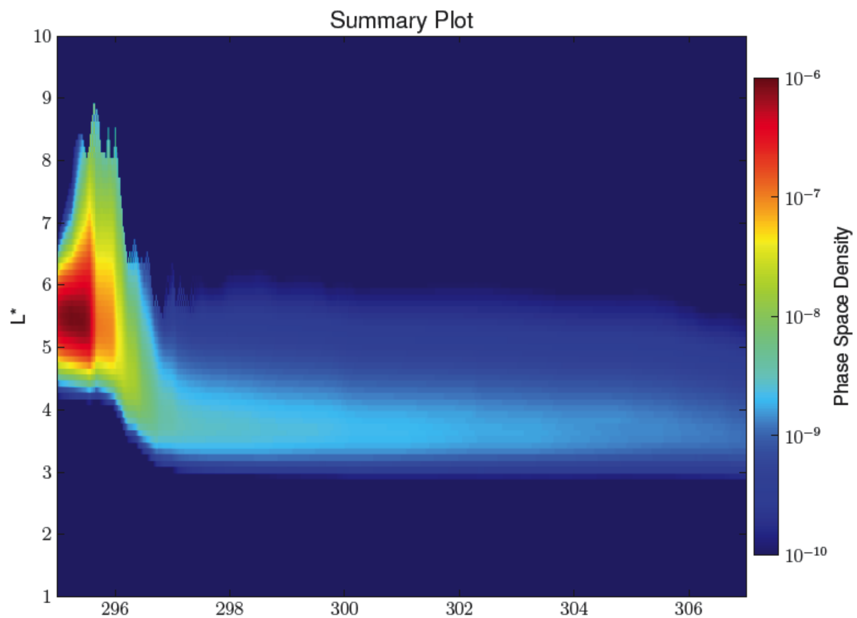
Here with data assimilation using enKF

missing acceleration term in physics model

a fixed model error is not representing the real model error

additional inflation and spreading in the ensemble is necessary otherwise ensemble diverges



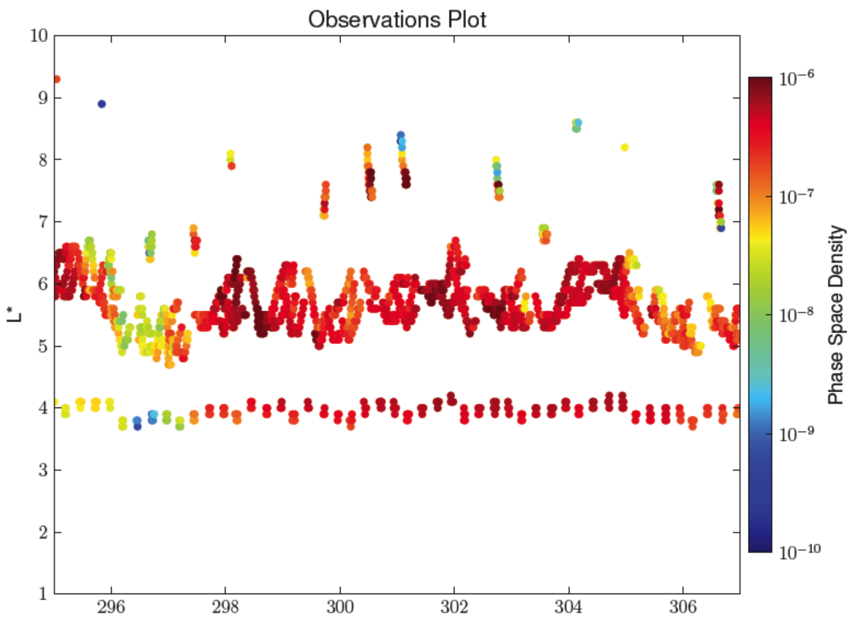


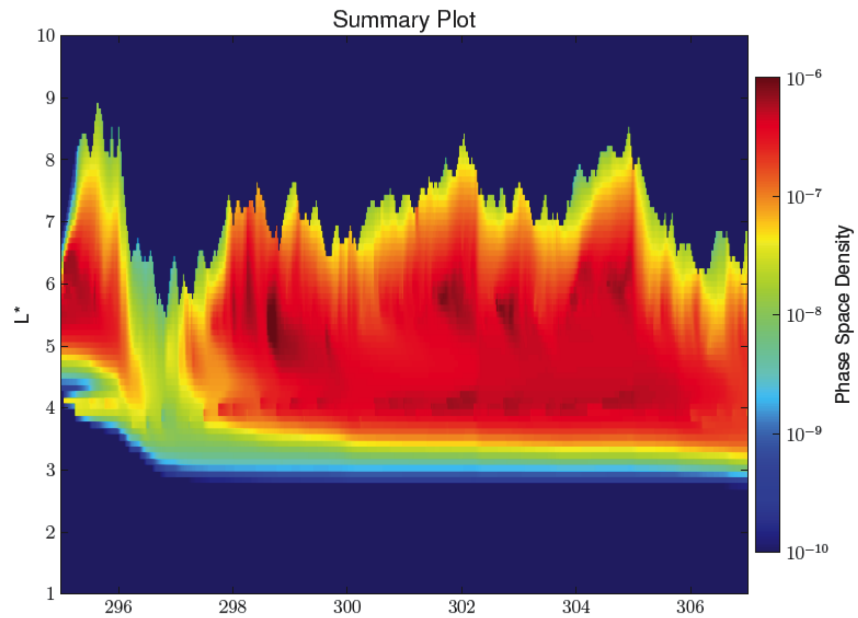
Two different inflation techniques

1. Inflate ensemble by adaptively adding white noise to the model state to compensate for missing source term

2. Add bias to ensemble

These will enable the enKF to guide the ensemble towards the observations

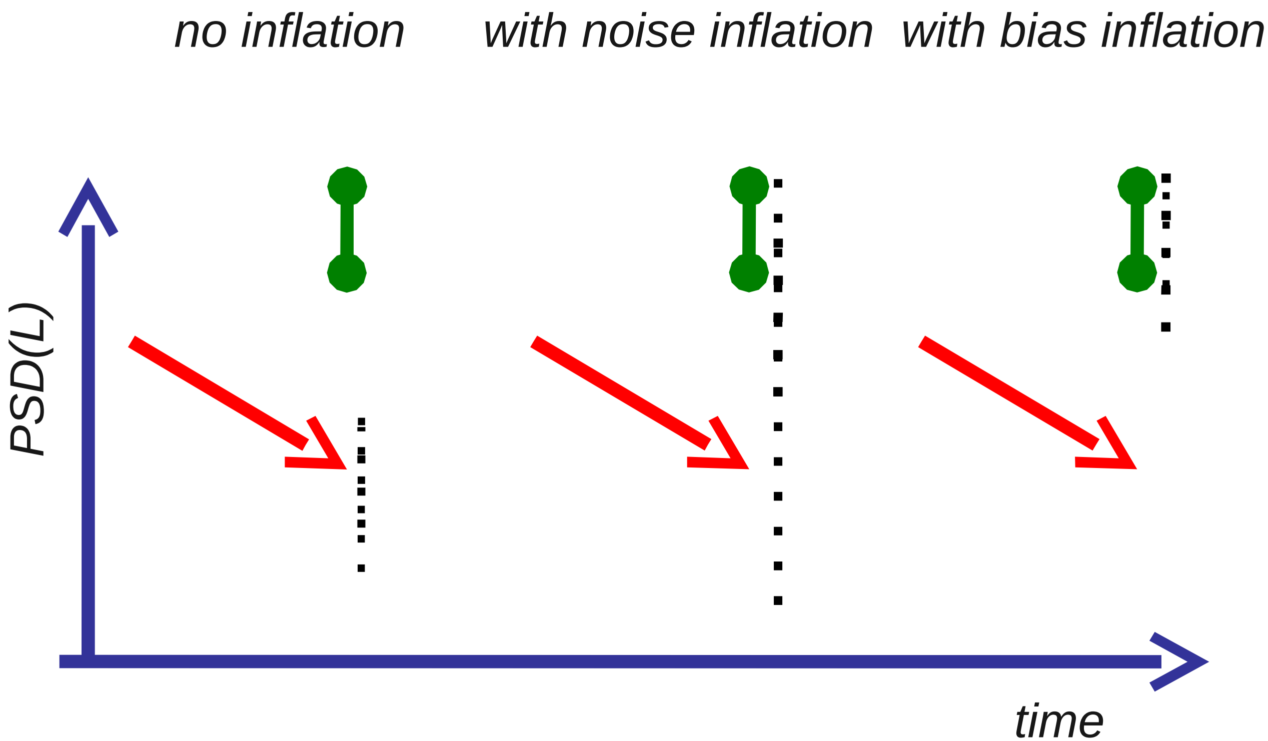




Inflation Methods

applying and inflation method is key to compensate for missing physics

Bias inflation is likely to be the most appropriate



Summary

The DREAM data assimilation framework uses an ensemble Kalman Filter (enKF) for

radiation belt assimilation and research

solar magnetogram assimilation (joint LANL-AFOSR project)

Challenges:

Watch out for accurate error descriptions for data and model

If model is very wrong like 1D radiation belt diffusion without acceleration terms or special time varying boundary conditions, then: an error inflation method might be quite appropriate

Most of the algorithms are available in SpacePy

http://spacepy.lanl.gov