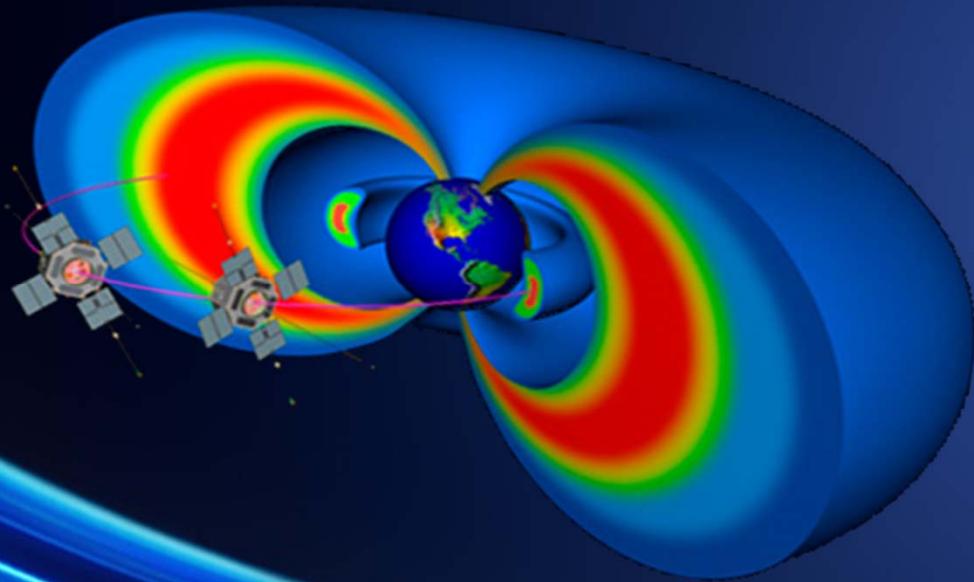


Quantitative Modeling of Radiation Belt Dynamics: Overview and Challenges



Weichao Tu

West Virginia University

2015/06/16

Special Thanks to:

J. M. Albert, W. Li, S. K. Morley,

G. S. Cunningham, Y. Chen, R.

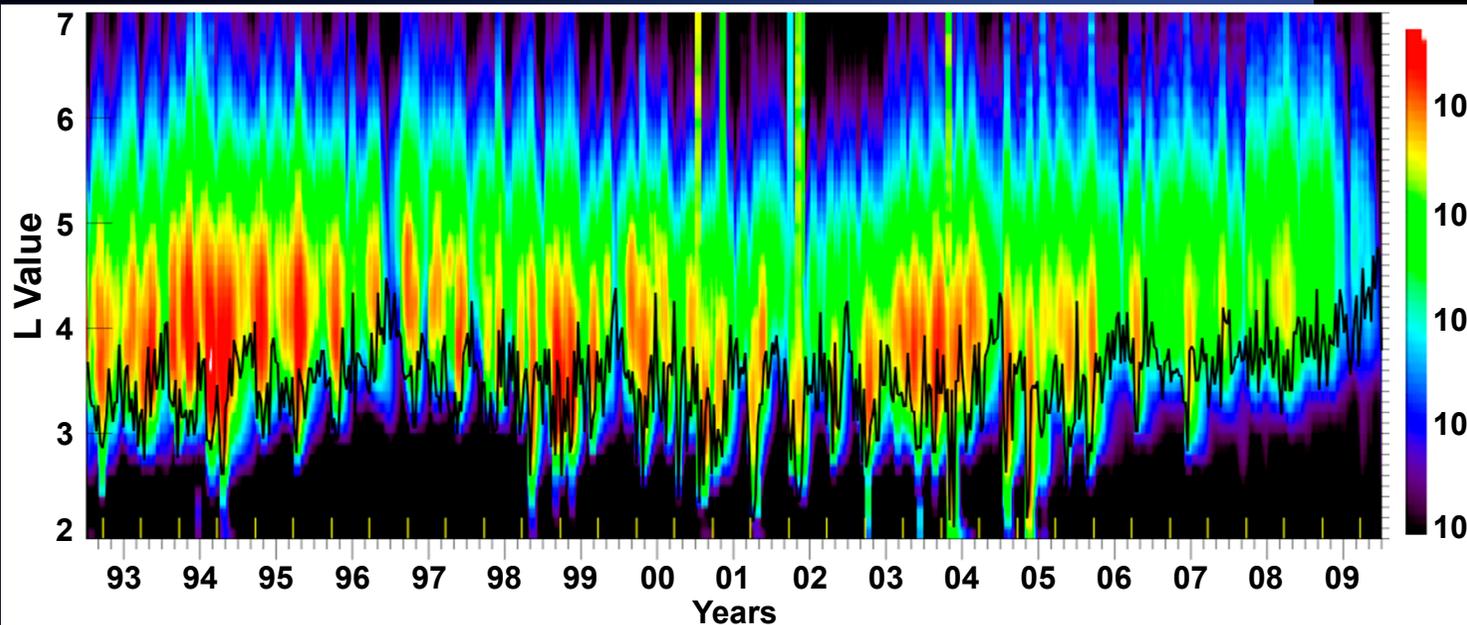
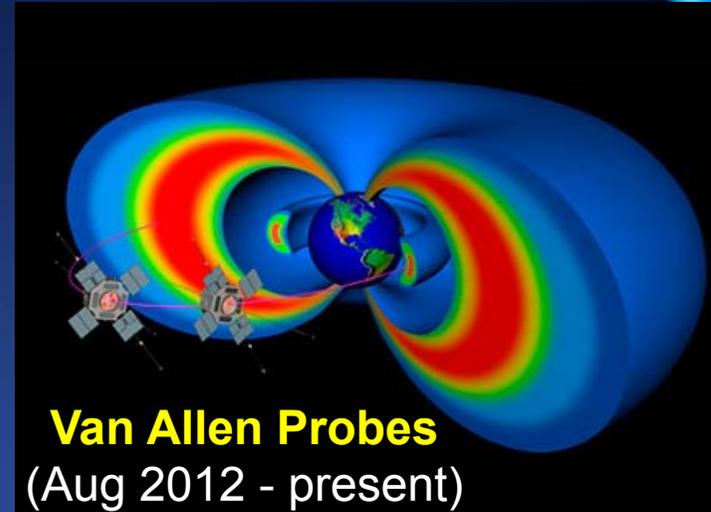
H. W. Friedel, G. D. Reeves, X. Li

Outline

- Introduction of Radiation Belt Dynamics: Sources and Losses
- Advances in Quantitative RB Modeling: Models and Inputs
- Challenges and Opportunities

Radiation Belt Dynamics

- Outer electron belt is very dynamic!
- The complex dynamics is a delicate balance of source, transport, and loss.
- Understanding the dynamics is the No.1 goal of the NASA Van Allen Probes Mission.

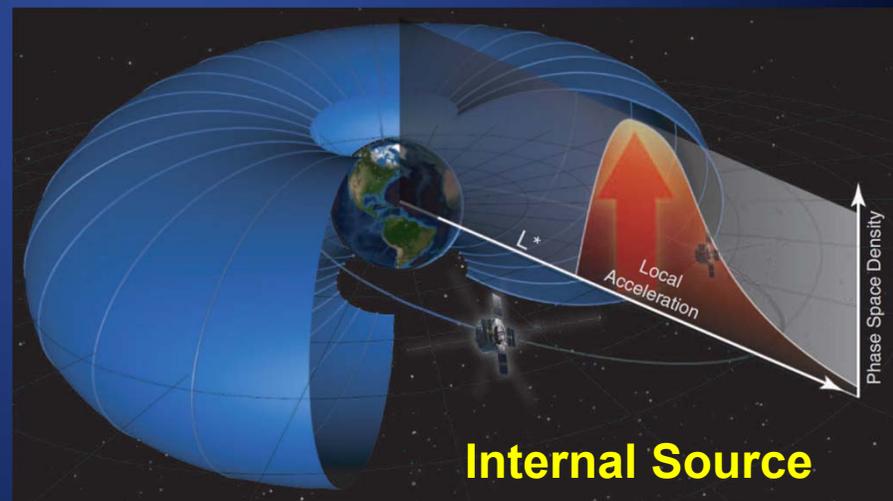
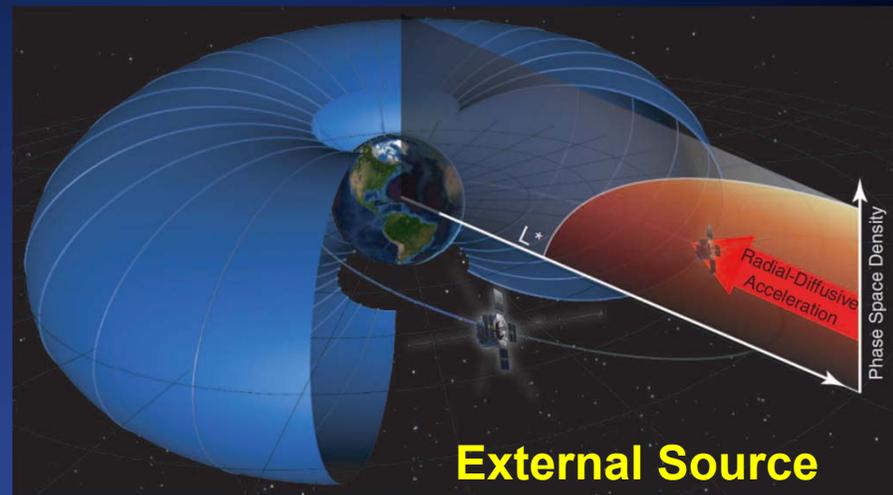
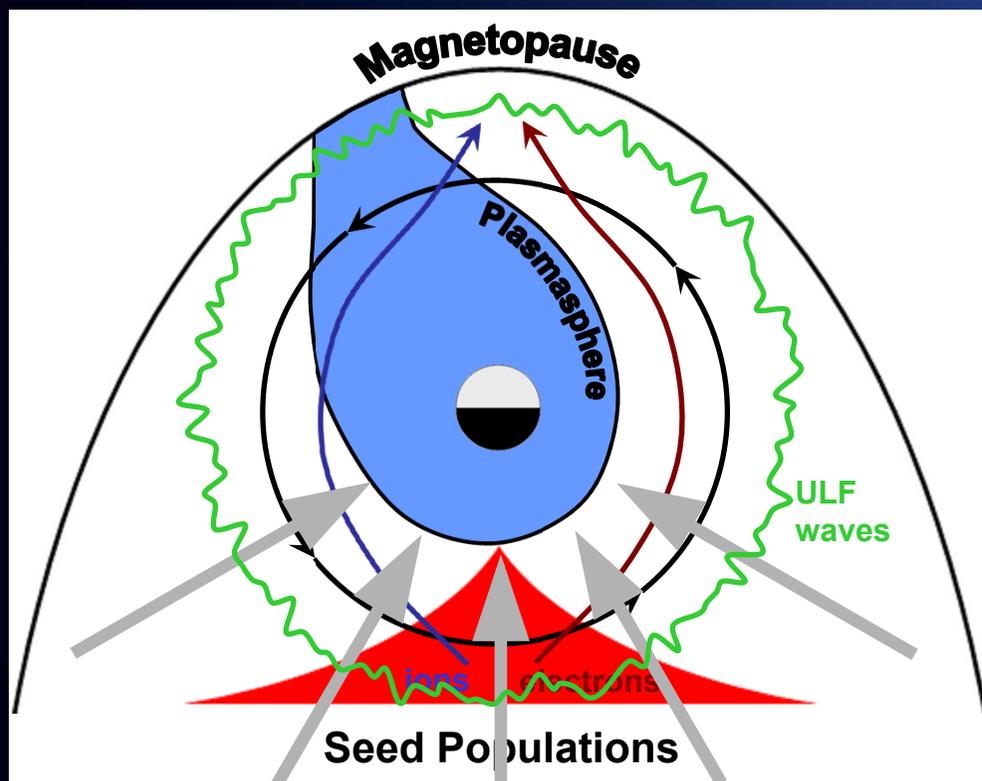


• Color-coded:
SAMPEX 2-6 MeV
Electron Flux (in
log).

[Courtesy of Xinlin Li]

Radiation Belt Dynamics: Sources

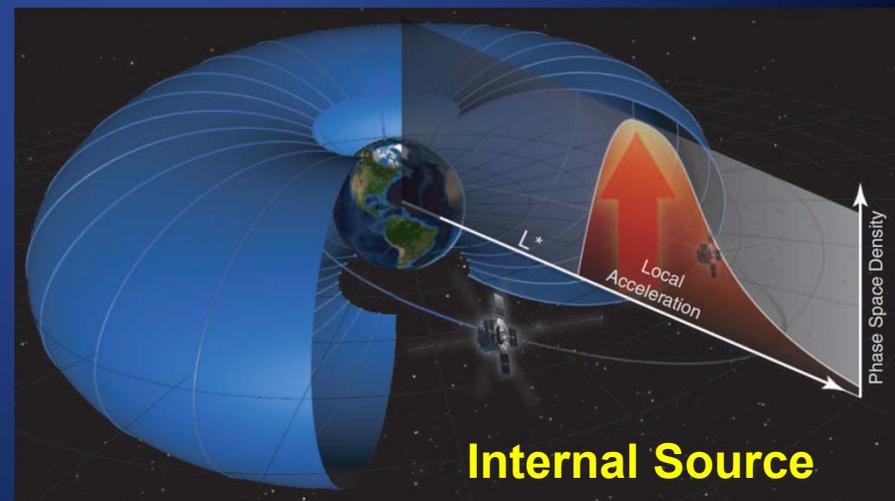
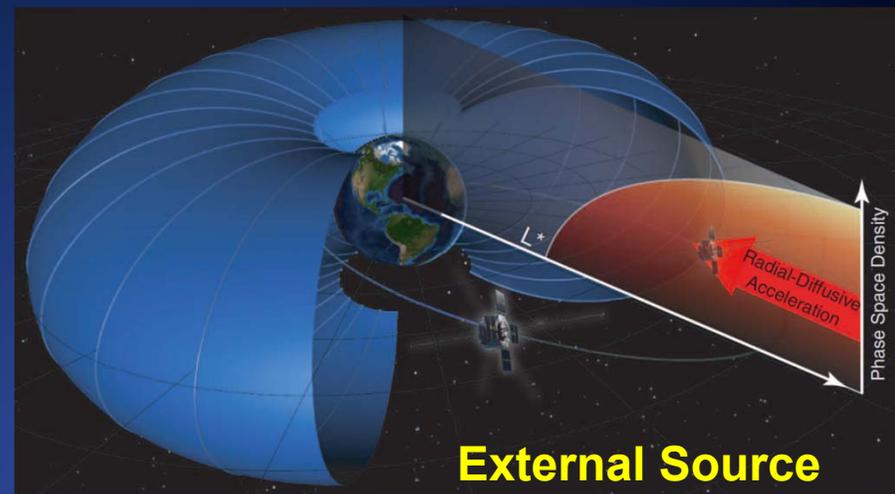
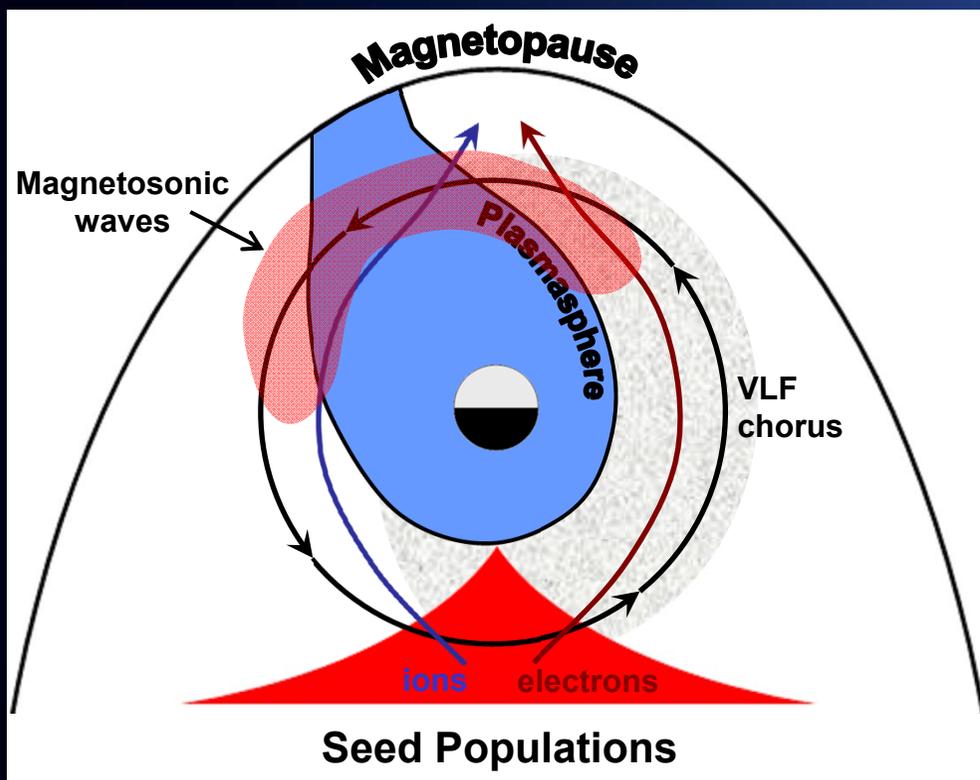
- **External source:** Inward radial transport due to interaction with **ULF waves**.



[Reeves et al., Science 2013]

Radiation Belt Dynamics: Sources

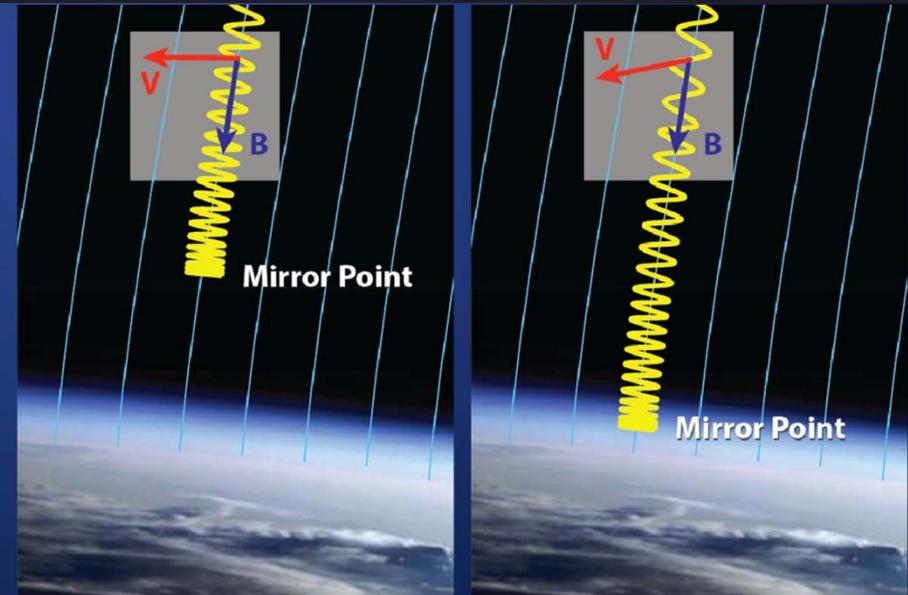
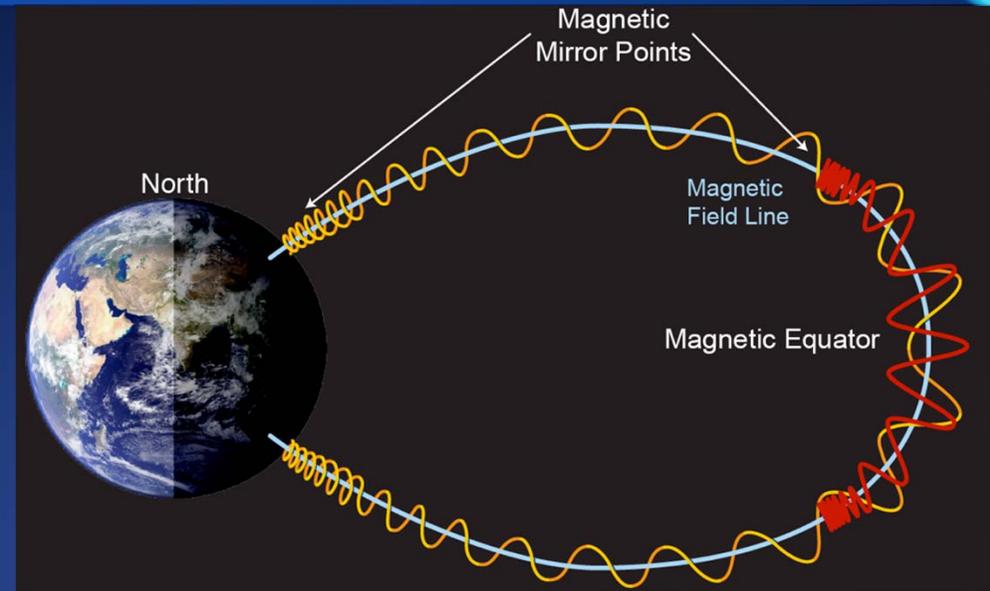
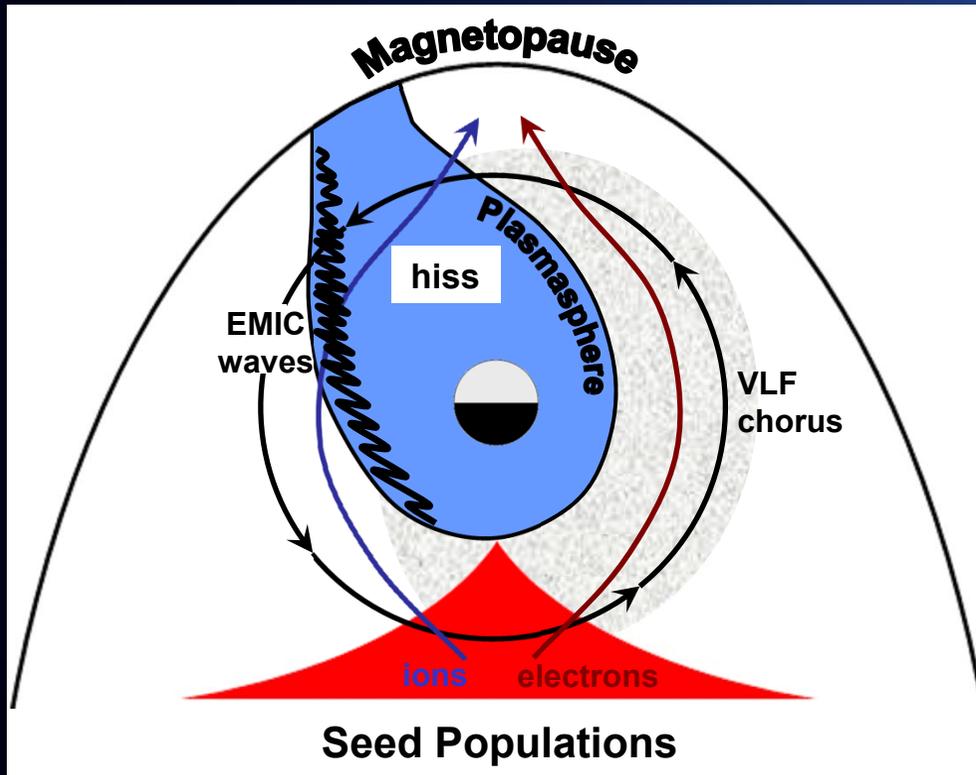
- **External source:** Inward radial transport due to interaction with **ULF waves**.
- **Internal source:** Local acceleration due to interaction with **VLF waves**.



[Reeves et al., Science 2013]

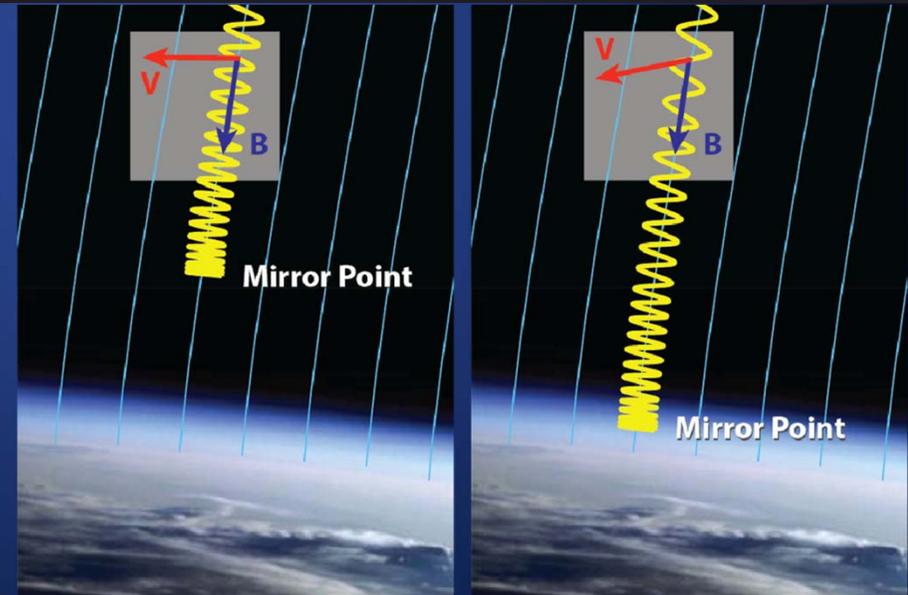
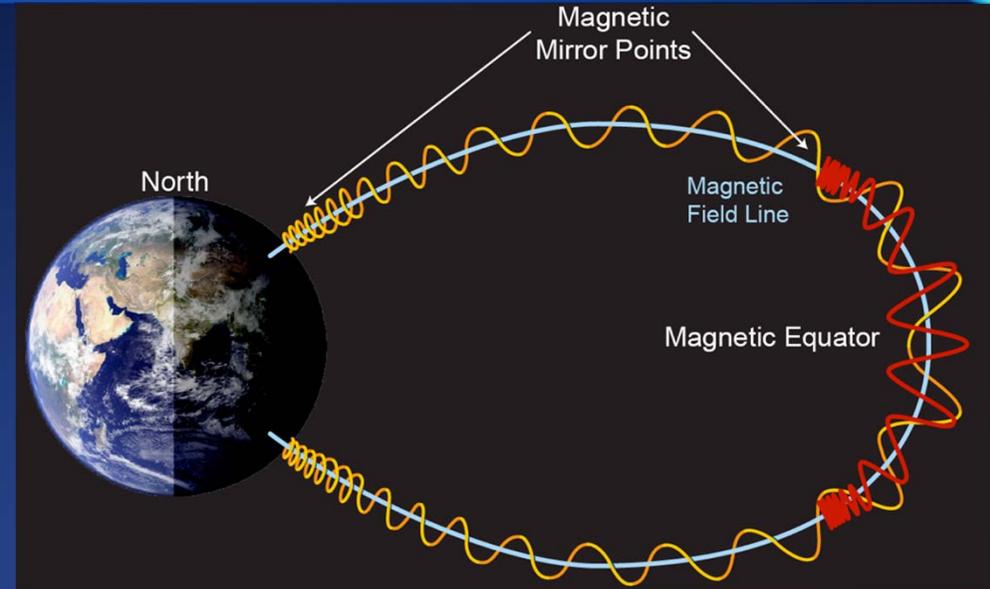
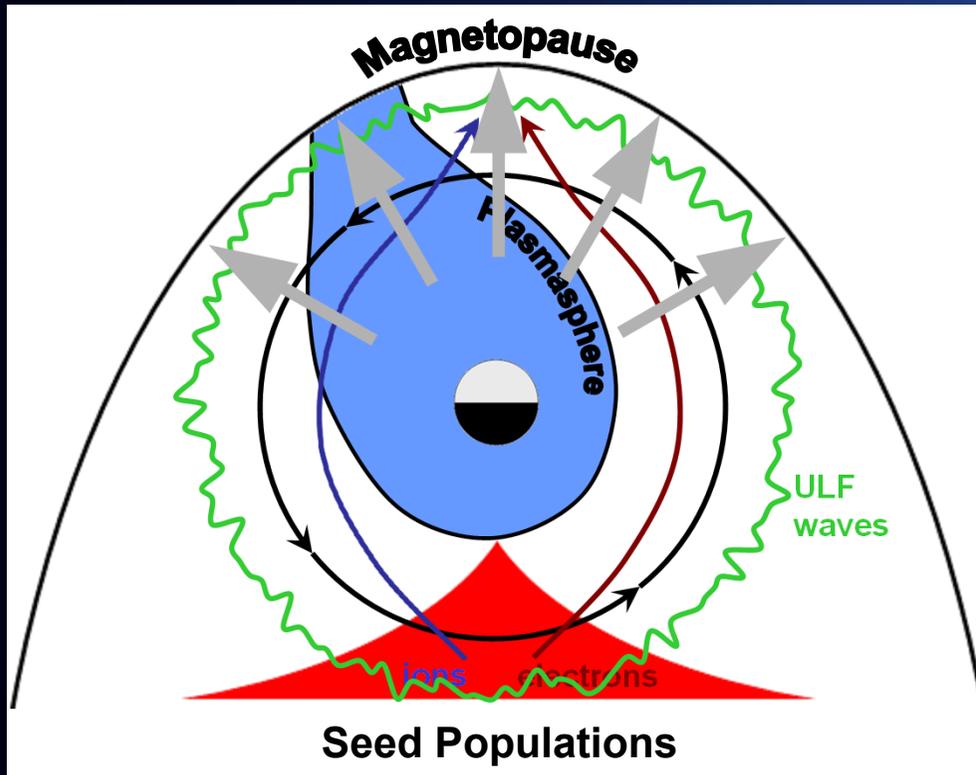
Radiation Belt Dynamics: Losses

- **Precipitation loss:** pitch angle scattering by, e.g., **VLF, EMIC waves**.



Radiation Belt Dynamics: Losses

- **Precipitation loss:** pitch angle scattering by, e.g., VLF, EMIC waves.
- **Magnetopause shadowing:** outward radial transport (ULF waves) or magnetopause compression.

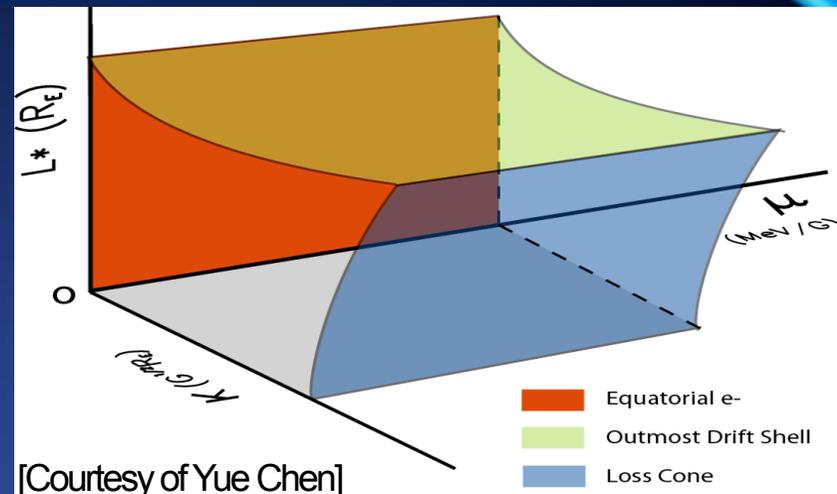


Outline

- Introduction of Radiation Belt Dynamics: Sources and Losses
- Advances in Quantitative RB Modeling: Models and Inputs
- Challenges and Opportunities

Types of Radiation Belt Models

- Fokker-Planck diffusion models
- Convection-Diffusion models
- Test particle codes
- PIC & Hybrid codes



Inputs

Diffusion coefficients
Boundary conditions

Outputs

PSD(μ, K, L, time)

Pros:

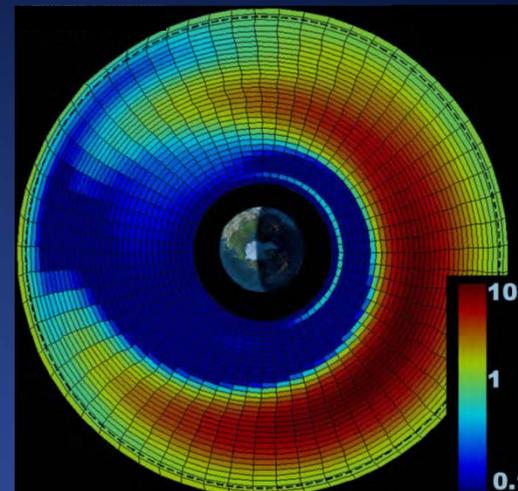
- Efficient, useful for global problems

Cons:

- Assumptions of quasi-linear theory and diffusion physics

Types of Radiation Belt Models

- Fokker-Planck diffusion models
- **Convection-Diffusion models**
- Test particle codes
- PIC & Hybrid codes



[Courtesy of V. Jordanova]

Inputs

Diffusion coefficients
Global E&B models

Pros:

- Include drift phase and convection physics, useful for seed population

Outputs

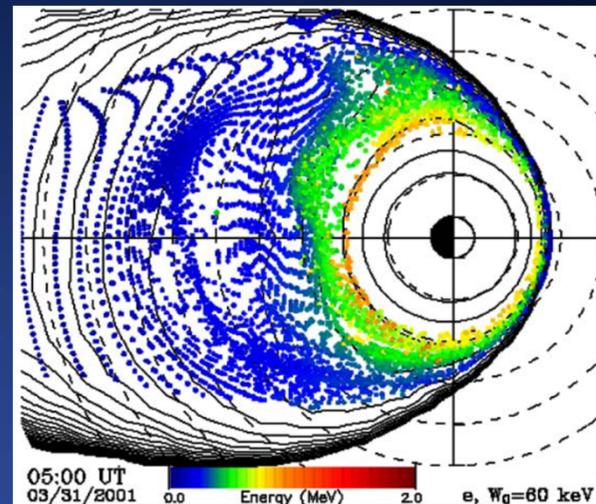
PSD($E, \alpha, r, \Phi, \text{time}$)

Cons:

- Assumptions of quasi-linearity and diffusion; performance of the global E&B models

Types of Radiation Belt Models

- Fokker-Planck diffusion models
- Convection-Diffusion models
- **Test particle codes**
- PIC & Hybrid codes



[Elkington et al., JASTP 2004]

Inputs

Global E&B models or
Analytical wave models



Outputs

$\text{flux}(E, \alpha, r, \Phi, \text{time})$

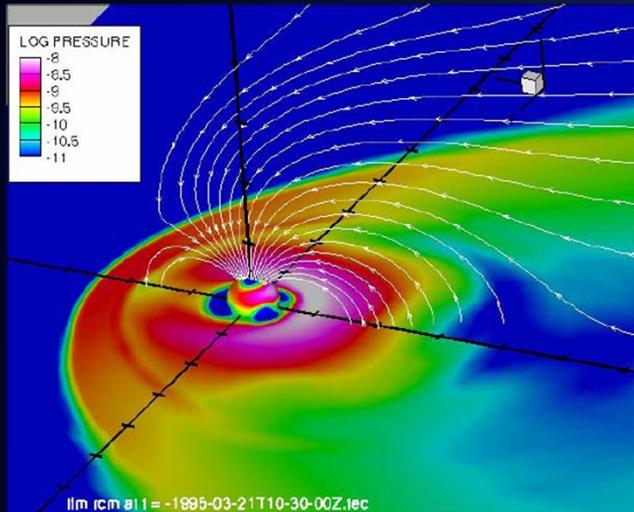
Pros:

- Include convection physics, no diffusion assumption

Cons:

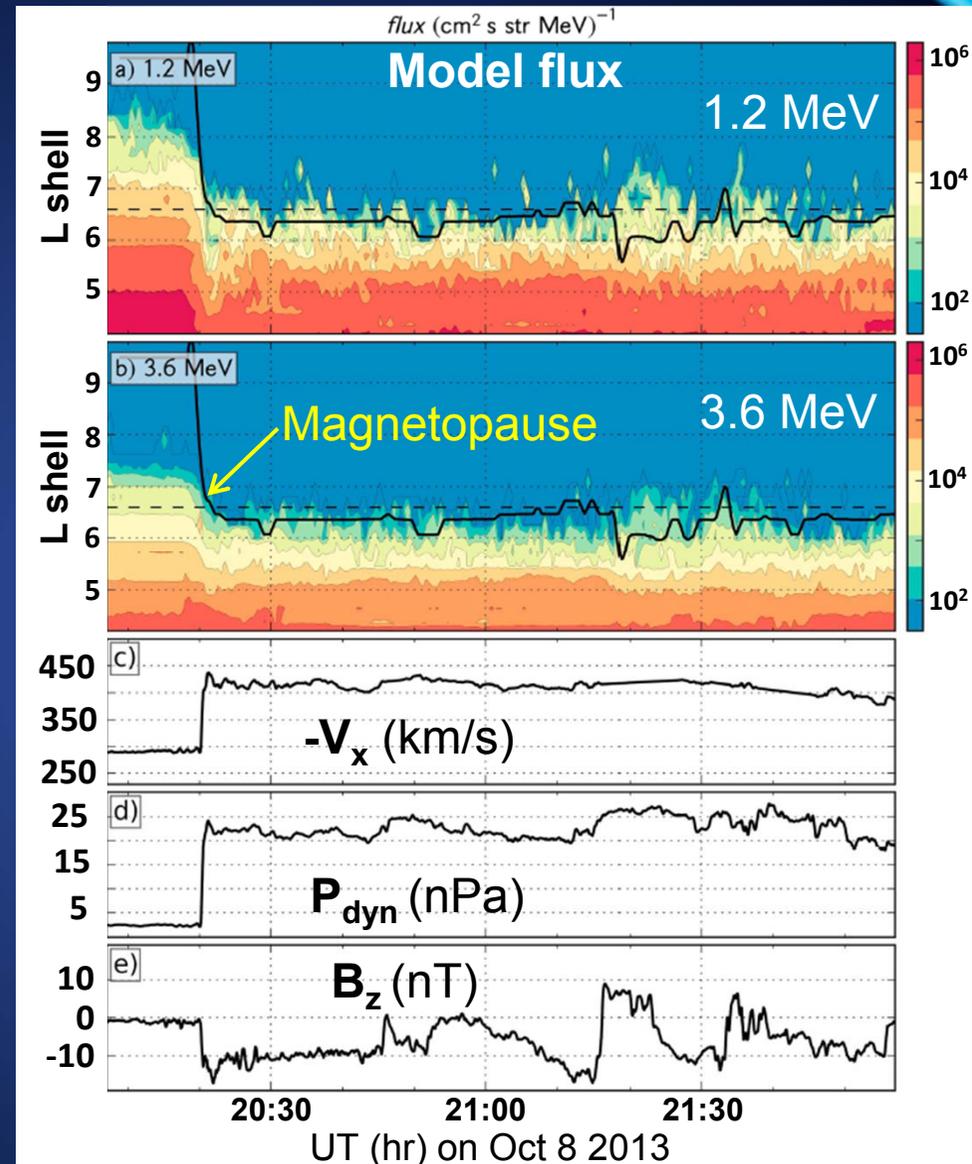
- Performance of the global E&B models (if MHD fields, no VLF waves); computationally expensive.

MHD Test Particle Simulation

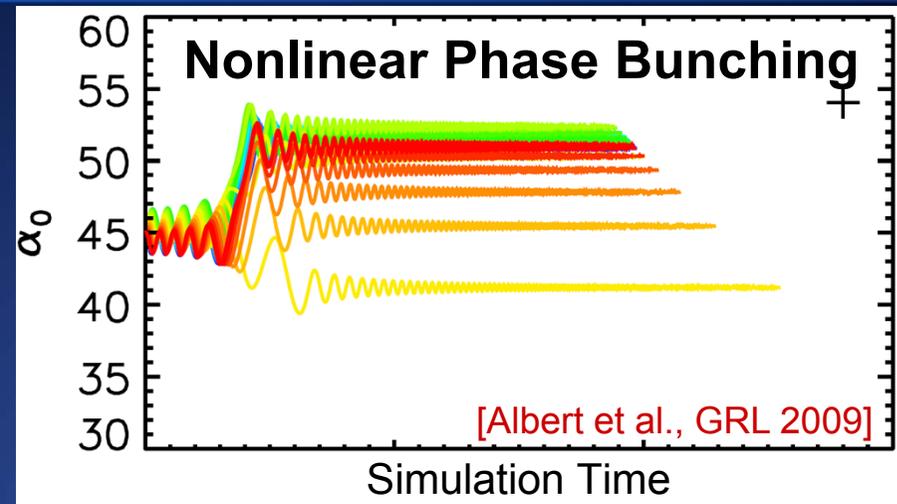
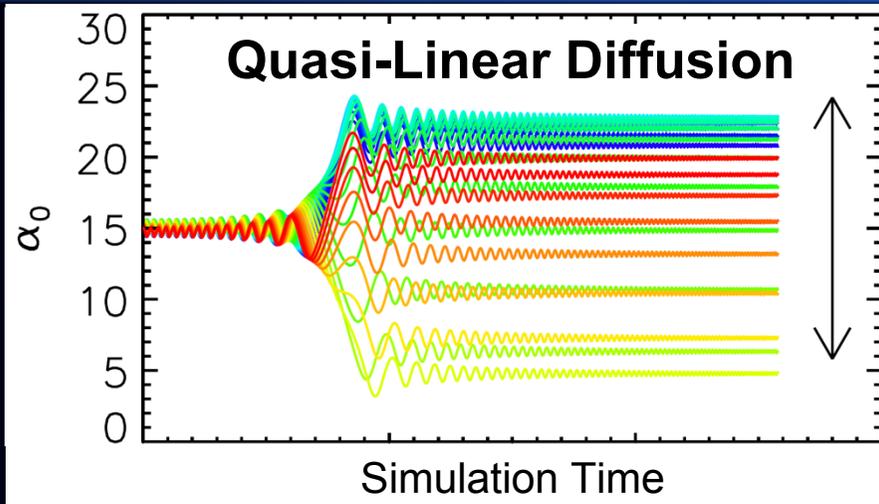


[Courtesy of F. Toffoletto]

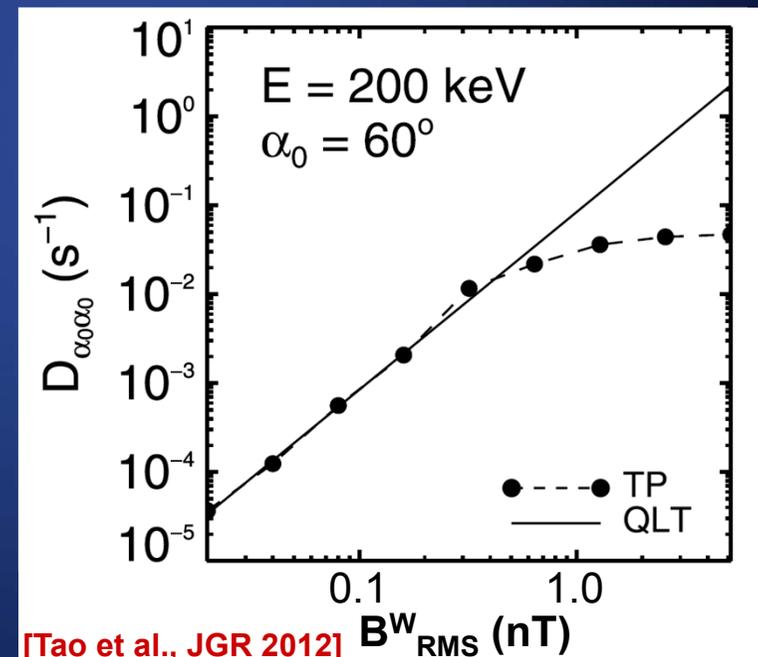
- Trace electrons under **global LFM-RCM fields** [Hudson et al., JGR 2015]
- **Loss of MeV electrons by magnetopause shadowing and outward radial transport (enhanced ULF waves)**



Test Quasi-Linear Theory

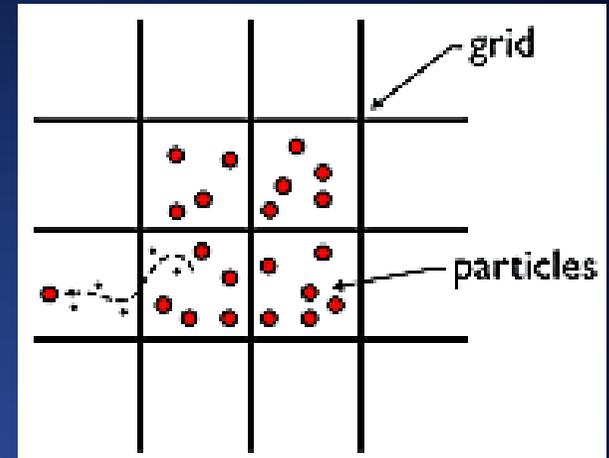


- Trace energetic electrons under a given wave model to **test quasi-linear theory and its limit**.
- E.g., Tao [JGR 2012] found that for parallel propagating whistler waves, **quasi-linear theory becomes invalid when the wave amplitude increases**.



Types of Radiation Belt Models

- Fokker-Planck diffusion models
- Convection-Diffusion models
- Test particle codes
- PIC & Hybrid codes



[from LANL website]

Inputs

Initial plasma and field conditions



Outputs

Wave growth and interaction

Pros:

- Self-consistent wave particle interactions

Cons:

- Computationally expensive; limited coupling to global codes

Types of Radiation Belt Models

- Fokker-Planck diffusion models
- Convection-Diffusion models
- Test particle codes
- PIC & Hybrid codes

Self-consistency Global efficiency



Inputs

Initial plasma and field conditions



Outputs

Wave growth and interaction

Pros:

- Self-consistent wave particle interactions

Cons:

- Computationally expensive; limited coupling to global codes

Fokker-Planck Diffusion Models

- **3D Fokker-Planck Equation** [Schulz and Lanzerotti, 1974]:

$$\begin{aligned} \frac{\partial f}{\partial t} = & L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) + \frac{1}{G} \frac{\partial}{\partial K} \left(GD_{KK} \frac{\partial f}{\partial K} \right) + \frac{1}{G} \frac{\partial}{\partial \mu} \left(GD_{\mu\mu} \frac{\partial f}{\partial \mu} \right) \\ & + \frac{1}{G} \frac{\partial}{\partial K} \left(GD_{K\mu} \frac{\partial f}{\partial \mu} \right) + \frac{1}{G} \frac{\partial}{\partial \mu} \left(GD_{\mu K} \frac{\partial f}{\partial K} \right) \end{aligned}$$

- Simplified to **1D radial diffusion model** at fixed μ and K :

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) - \frac{f}{\tau} + S$$

τ : electron lifetime from pitch angle diffusion;
 S : electron source rate from energy diffusion

- Useful when radial diffusion is the dominant process.
- Need reliable inputs for D_{LL} , lifetime and source rate.

1D Radial Diffusion Model

- [Tu et al., JGR 2009]

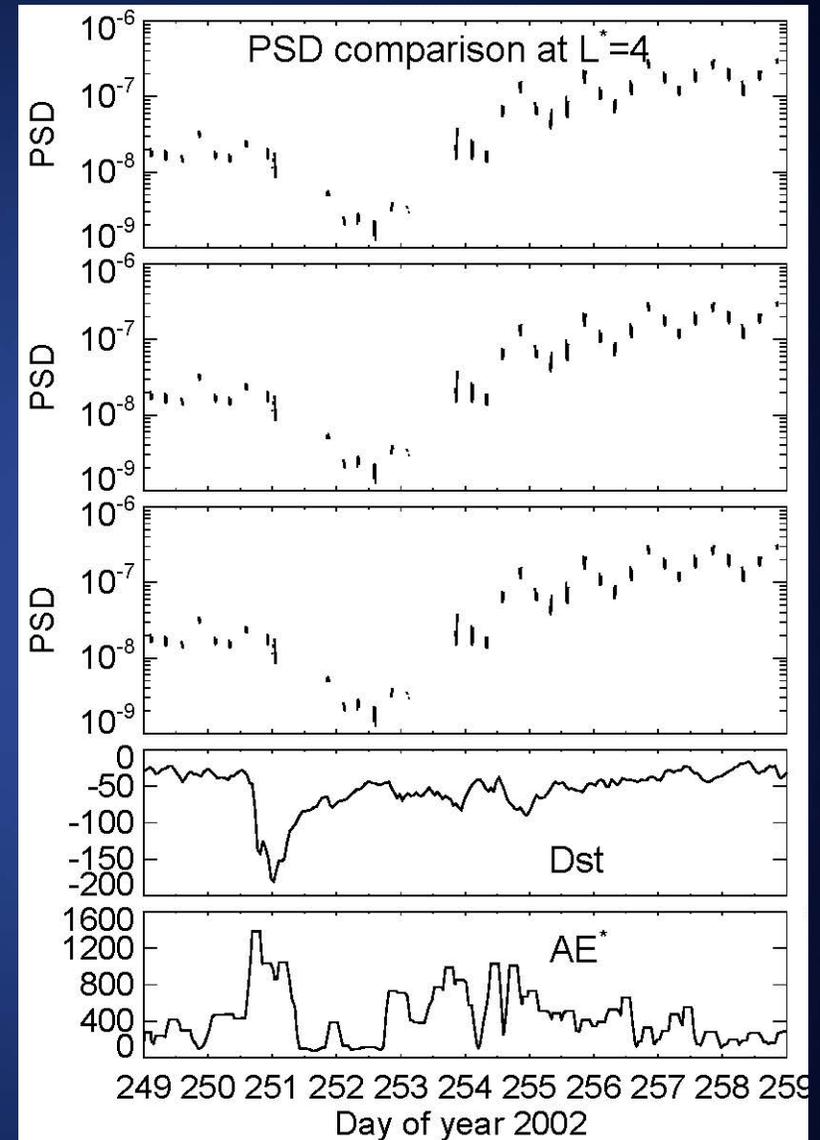
$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) - \frac{f}{\tau} + S$$

- Goal:** to reproduce the PSD dynamics observed at L=4 (with data-driven outer boundary at L=6)

- Empirical inputs:**

$$D_{LL}(Kp, D_0) = D_0 \times 10^{(0.506Kp - 9.325)} L^{10}$$

$$S(AE, S_0) \text{ and } \tau(AE, Dst, \tau_0)$$

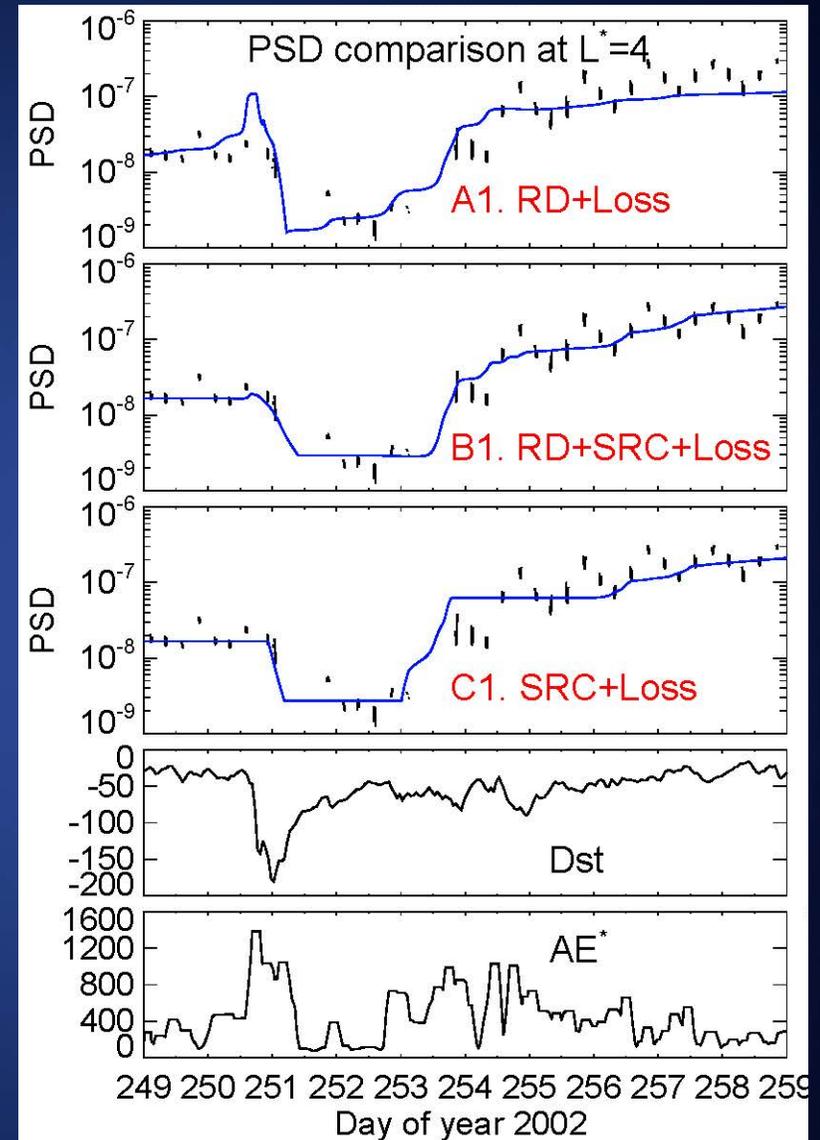


1D Radial Diffusion Model

- [Tu et al., JGR 2009]

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) - \frac{f}{\tau} + S$$

- Goal:** to reproduce the PSD dynamics observed at L=4 (with data-driven outer boundary at L=6)
- Empirical inputs:**
 $D_{LL}(Kp, D_0) = D_0 \times 10^{(0.506Kp - 9.325)} L^{10}$
 $S(AE, S_0)$ and $\tau(AE, Dst, \tau_0)$
- Model results:** well-reproduce the dynamics by all three different runs.
- Uncertainties in the model inputs:** D_{LL} , electron lifetime, and source rate

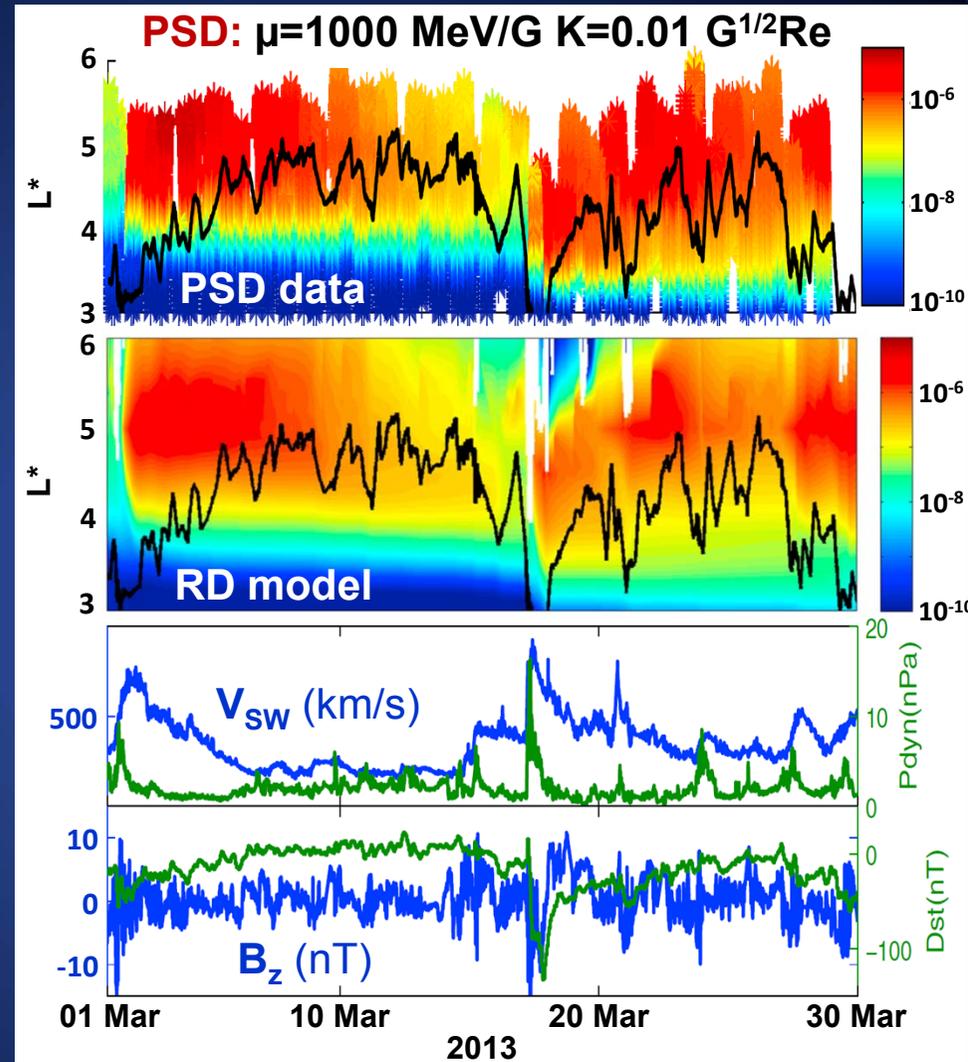


1D Radial Diffusion Model

- Z. Li et al. [JGR 2014] simulated the **1-month Van Allen Probes interval in March 2013**.

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) - \frac{f}{\tau}$$

- The 1st enhancement at the beginning of March is reproduced by radial diffusion from a source at larger L .
- The Mar 17 storm enhancement is under-reproduced by radial diffusion only.



2D Diffusion Model

- Coordinates
(μ, K, L)



(p, α, L)

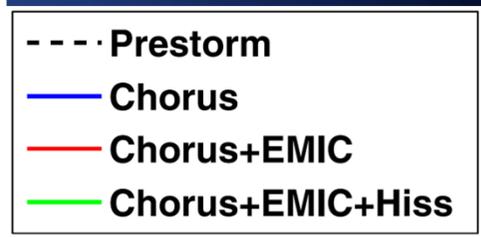
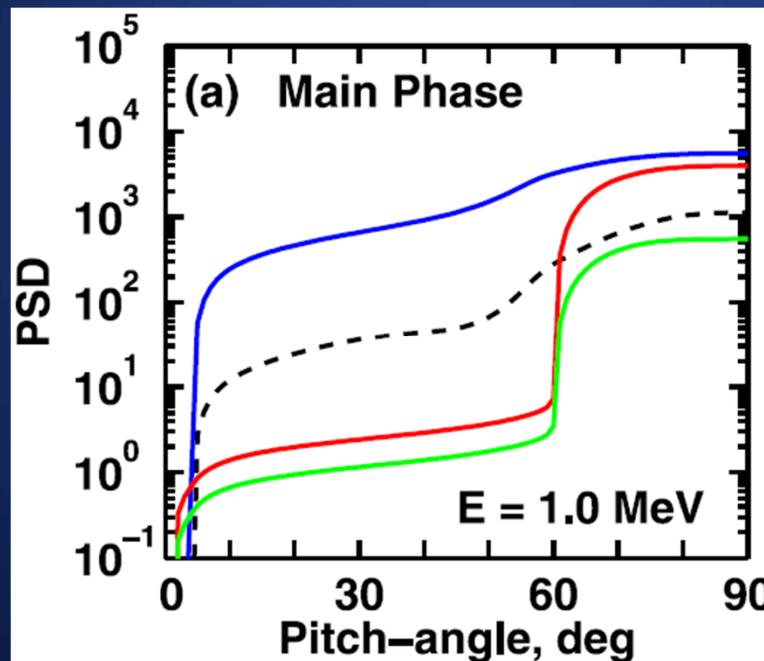
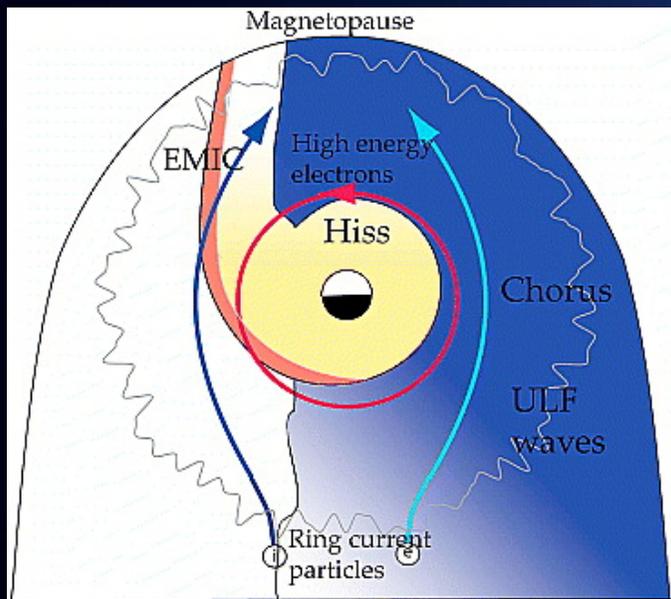
$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) + \frac{1}{G} \frac{\partial}{\partial K} \left(GD_{KK} \frac{\partial f}{\partial K} \right) + \frac{1}{G} \frac{\partial}{\partial \mu} \left(GD_{\mu\mu} \frac{\partial f}{\partial \mu} \right) + \frac{1}{G} \frac{\partial}{\partial K} \left(GD_{K\mu} \frac{\partial f}{\partial \mu} \right) + \frac{1}{G} \frac{\partial}{\partial \mu} \left(GD_{\mu K} \frac{\partial f}{\partial K} \right)$$

$$\frac{\partial f}{\partial t} = \frac{1}{\Gamma} \frac{\partial}{\partial \alpha} \left(\Gamma D_{\alpha\alpha} \frac{\partial f}{\partial \alpha} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial p} \right) + \frac{1}{\Gamma} \frac{\partial}{\partial \alpha} \left(\Gamma D_{\alpha p} \frac{\partial f}{\partial p} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{p\alpha} \frac{\partial f}{\partial \alpha} \right)$$

- Simplified to **2D momentum-pitch angle diffusion at fixed L**
- Useful to model **fast local acceleration and losses**, when radial diffusion is not dominant.

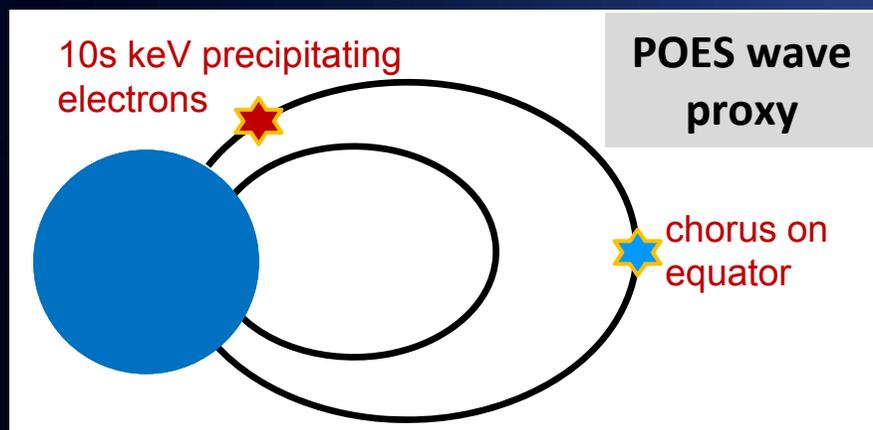
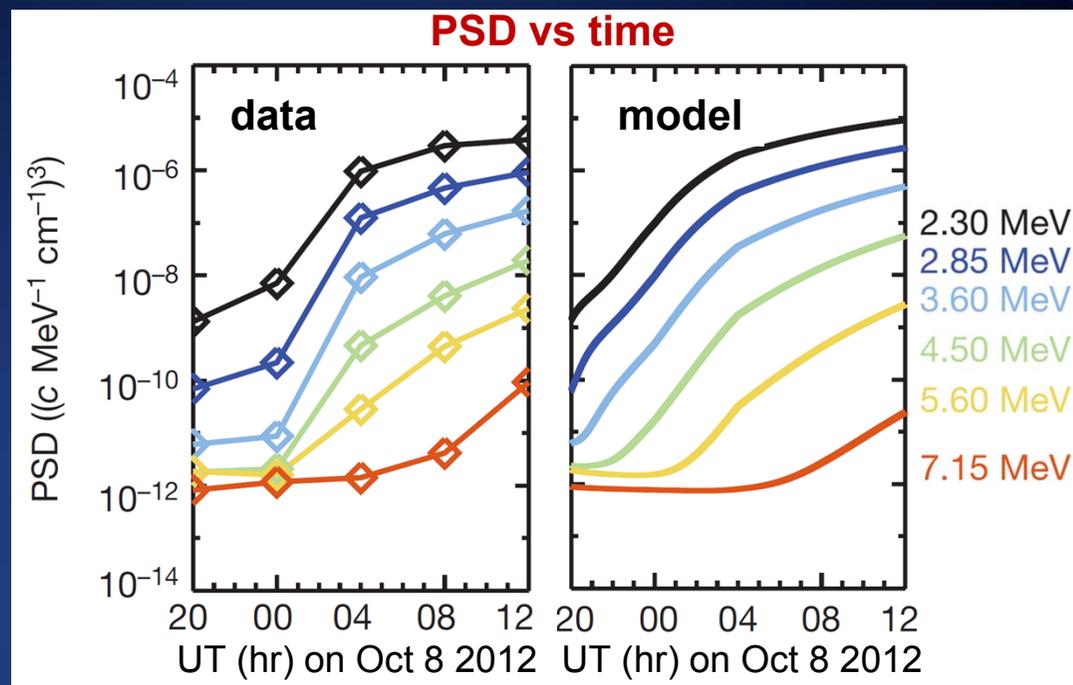
2D Diffusion Model

- [W. Li et al., JGR 2007]: Applied a 2D diffusion model to study the **acceleration and loss effects from different types of waves**, located at different local times.
- Diffusion coefficients are calculated from **static wave inputs** specified for **dayside/nightside chorus, EMIC, and hiss waves**.

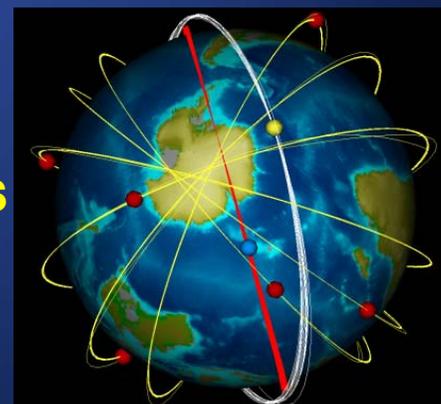


2D Diffusion Model

- Advance in model inputs:
 - [Thorne et al., Nature 2013]
 - Event-specific and global chorus wave model derived from POES proxy
 - Ambient plasma density derived from in situ data
- Model well-reproduced the strong RB enhancement up to 7.2 MeV



6 NOAA/POES



3D Diffusion Model

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) + \frac{1}{\Gamma} \frac{\partial}{\partial \alpha} \left(\Gamma D_{\alpha\alpha} \frac{\partial f}{\partial \alpha} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial p} \right) + \frac{1}{\Gamma} \frac{\partial}{\partial \alpha} \left(\Gamma D_{\alpha p} \frac{\partial f}{\partial p} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{p\alpha} \frac{\partial f}{\partial \alpha} \right)$$

- Many different versions have been developed in the past 10 years:
 - VERB (UCLA), DREAM3D (LANL), Dilbert? (Albert), REM (Rice), BAS model, Salammbô (French), STEERB (China) ...

- **Inputs:** Diffusion Coefficients

- **Boundary Conditions:**

$\alpha=0$ PSD=0 (atmosphere)

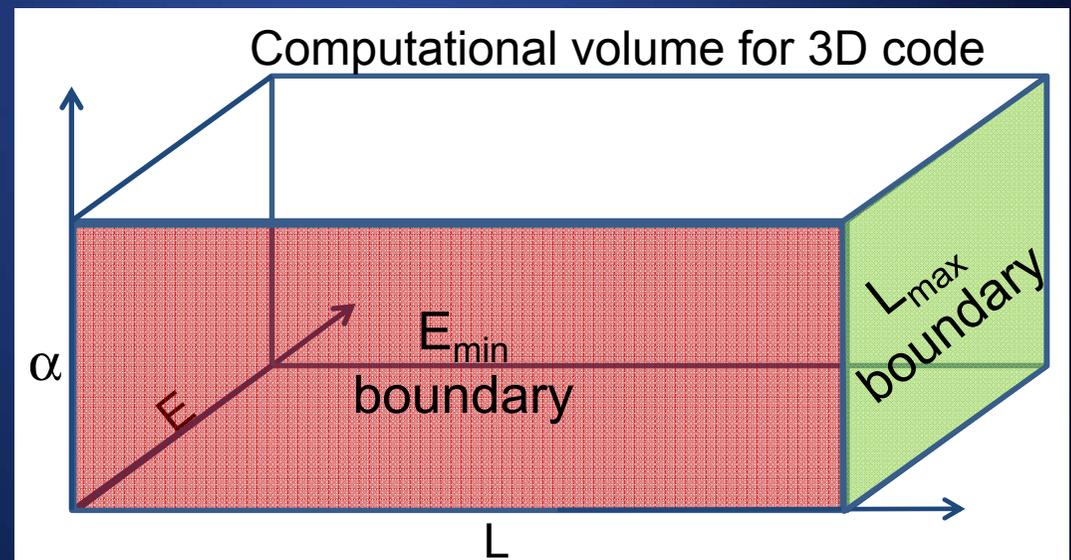
$\alpha=\pi/2$ $d\text{PSD}/d\alpha=0$

$L=1$ PSD=0 (atmosphere)

$L=L_{\max}$ **outer boundary**

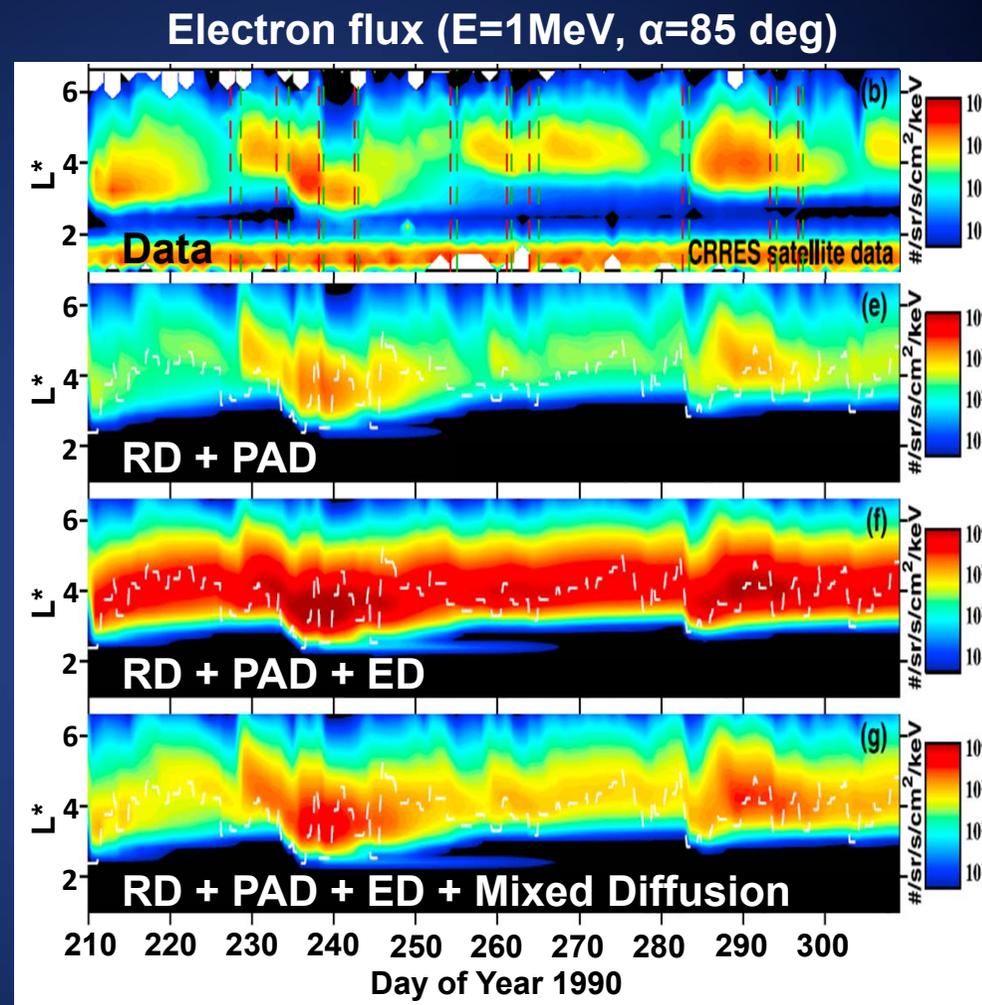
$E=E_{\max}$ PSD=0

$E=E_{\min}$ **seed population (100 KeV)**



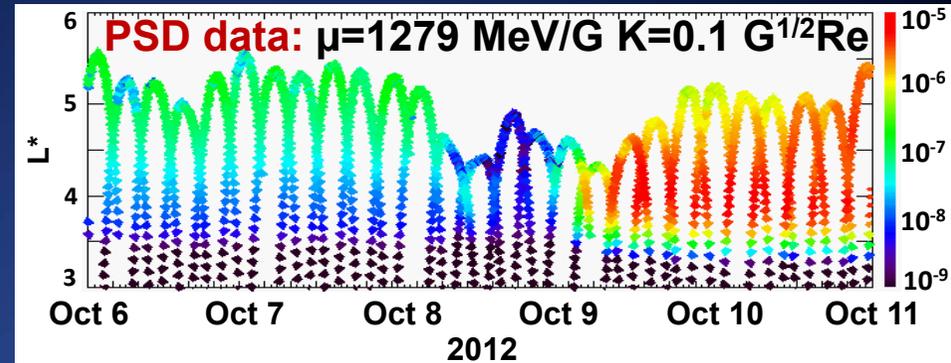
3D Diffusion Model

- VERB results from Subbotin et al. [JGR 2011]
- Long-term simulation of the CRRES interval to model the effects from different diffusion processes.
 - Best reproduce the observations by including all the diffusion processes
- Wave and plasma inputs: dynamic but still empirical (as a function of K_p).

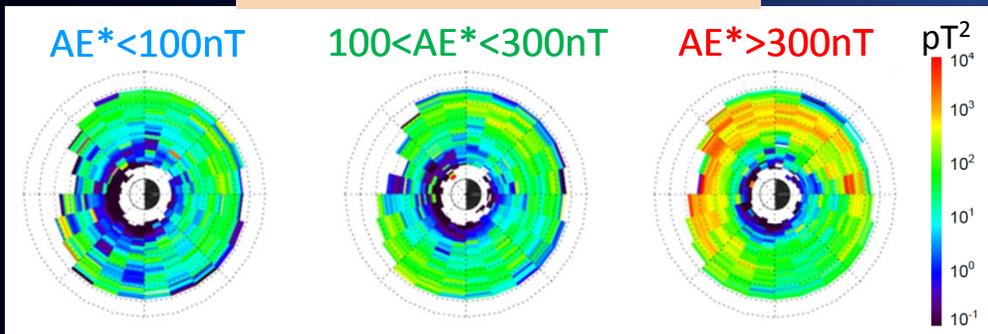


3D Diffusion Model

- Need event-specific inputs for very strong events.
- [Tu et al., GRL 2014]: DREAM3D
 - Model the strong enhancement during the Oct 2012 storm.



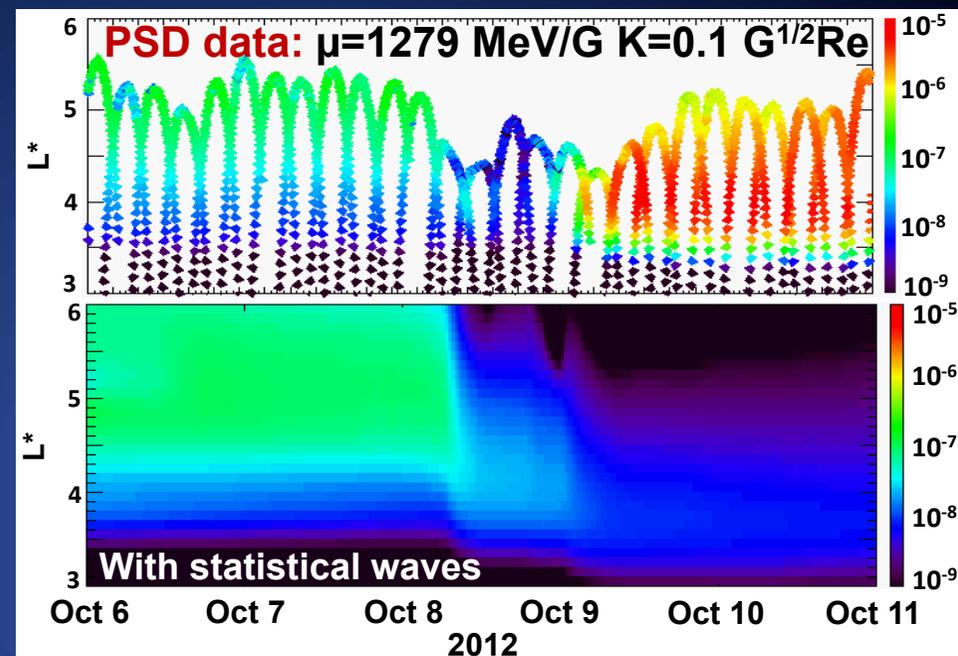
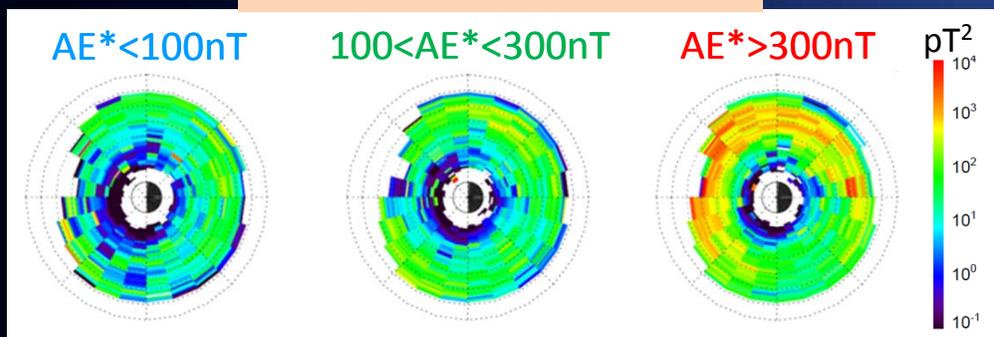
Statistical chorus model



3D Diffusion Model

- Need event-specific inputs for very strong events.
- [Tu et al., GRL 2014]: DREAM3D
 - Model the strong enhancement during the Oct 2012 storm.

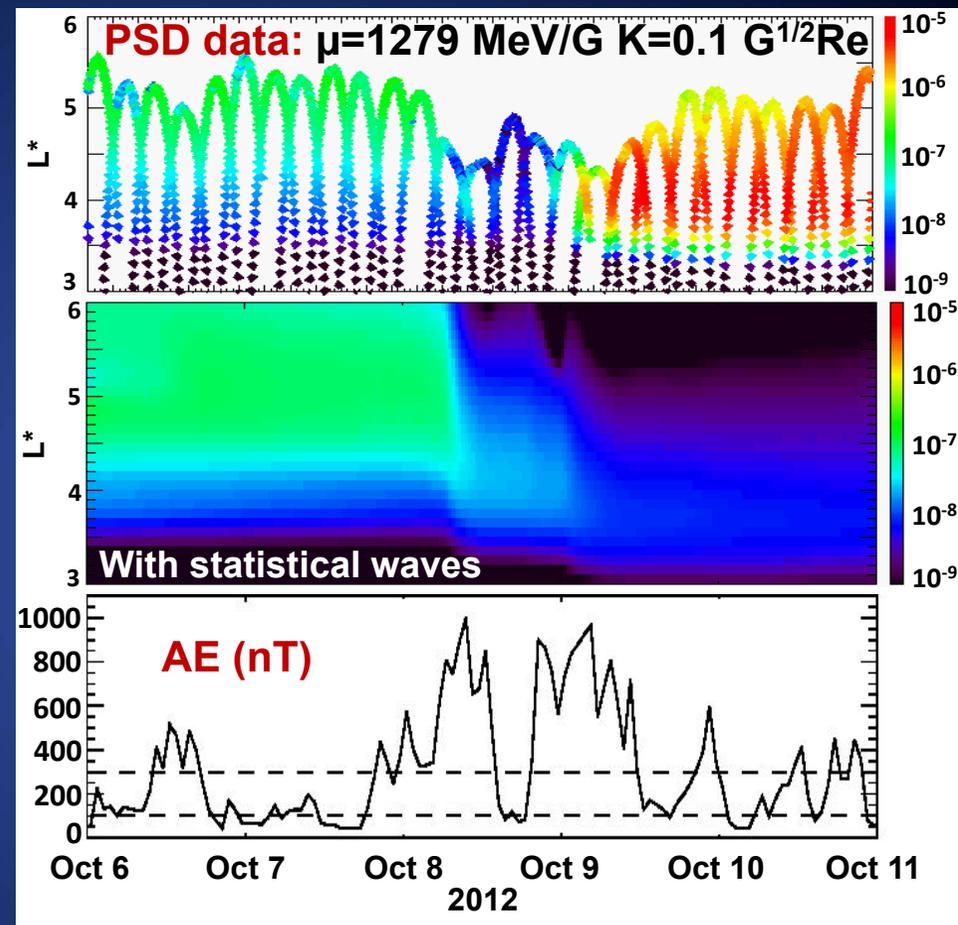
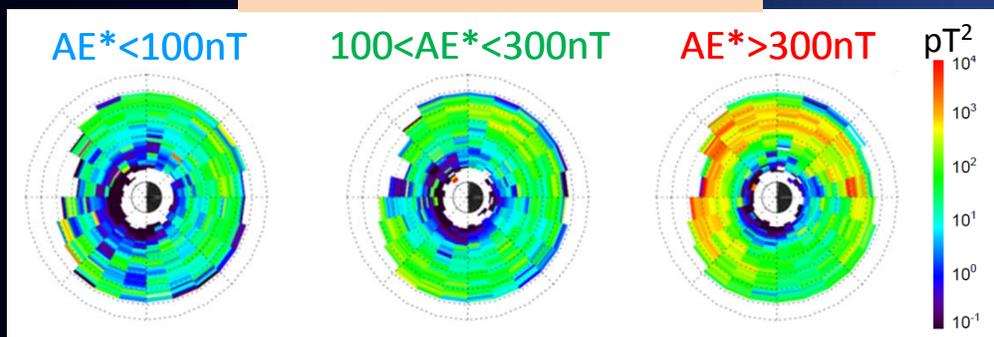
Statistical chorus model



3D Diffusion Model

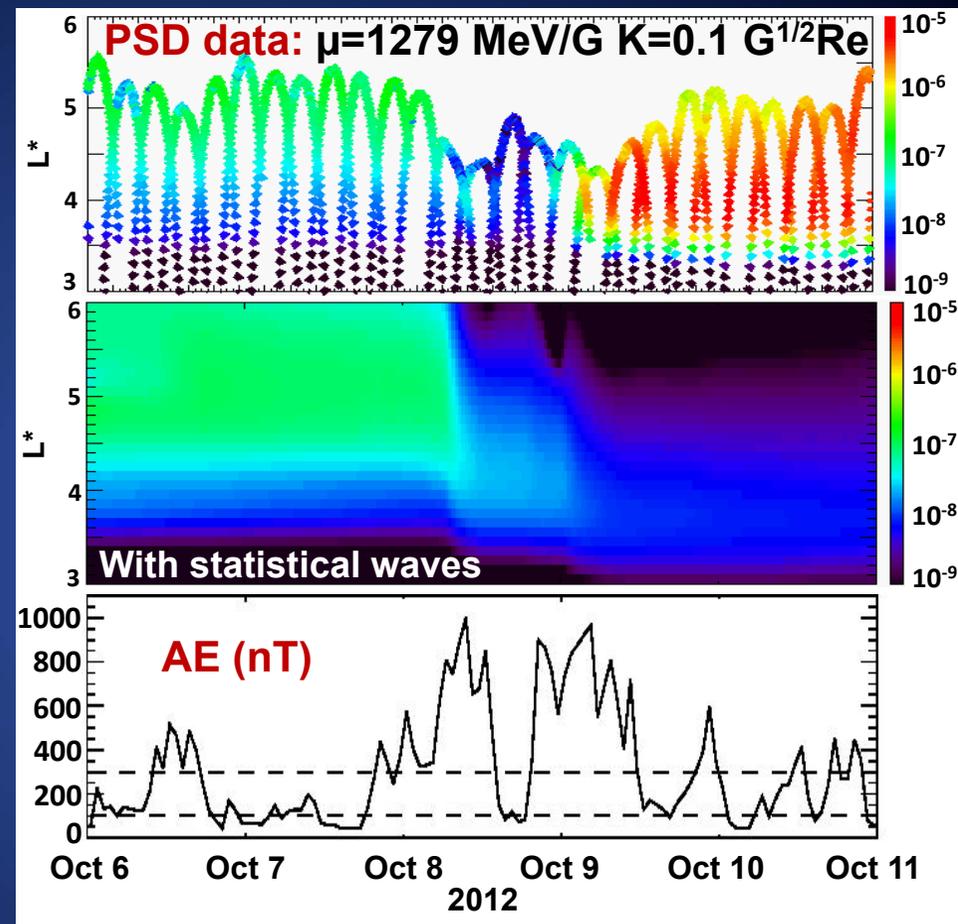
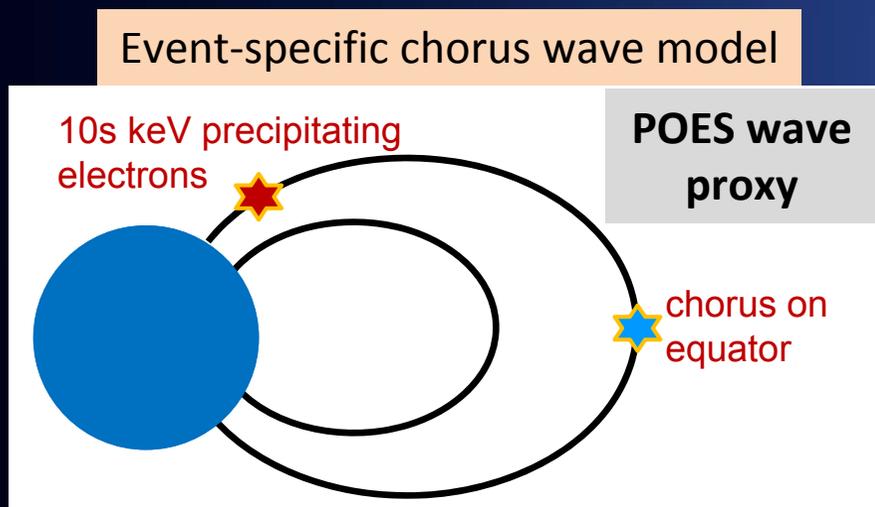
- Need event-specific inputs for very strong events.
- [Tu et al., GRL 2014]: DREAM3D
 - Model the strong enhancement during the Oct 2012 storm.

Statistical chorus model



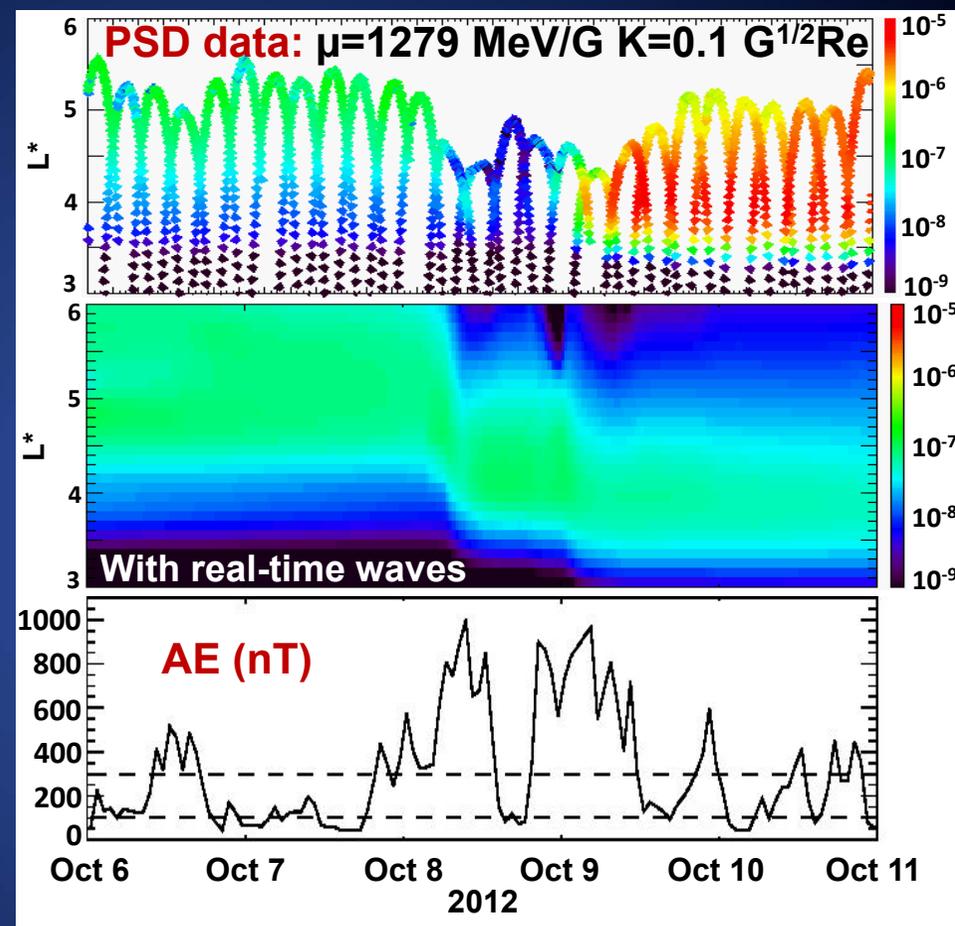
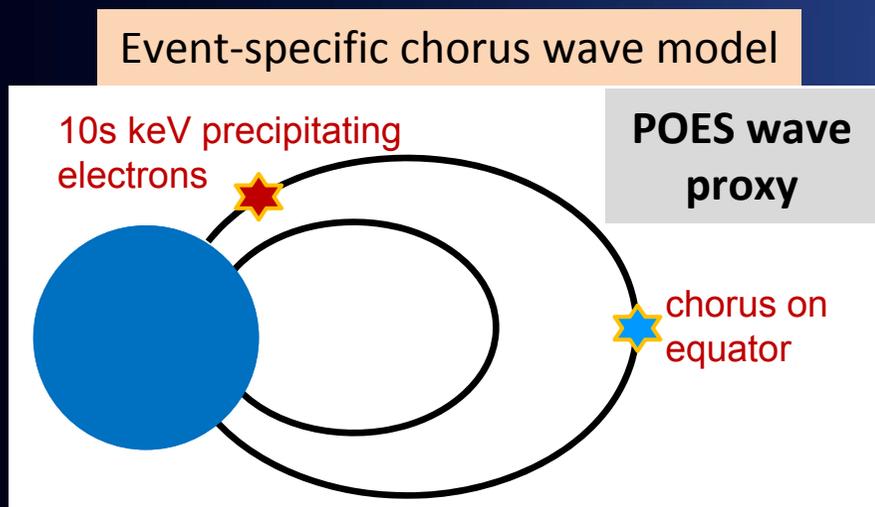
3D Diffusion Model

- Need event-specific inputs for very strong events.
- [Tu et al., GRL 2014]: DREAM3D
 - Model the strong enhancement during the Oct 2012 storm.
 - Event-specific chorus waves



3D Diffusion Model

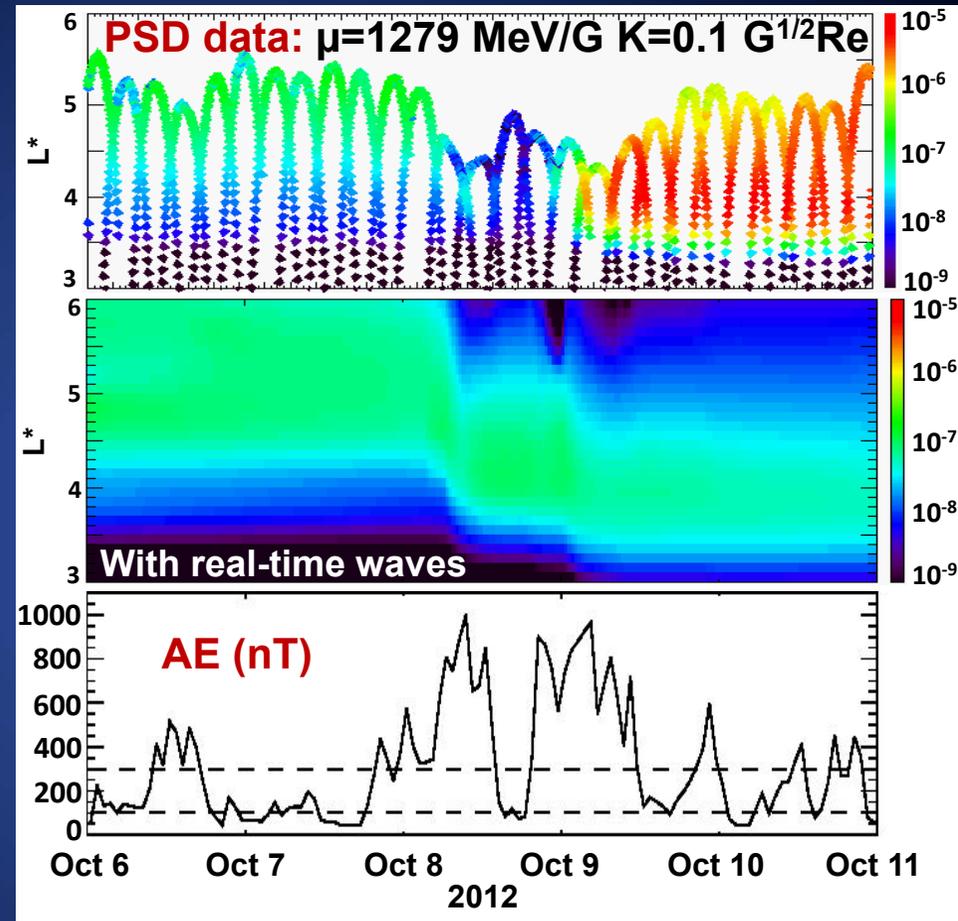
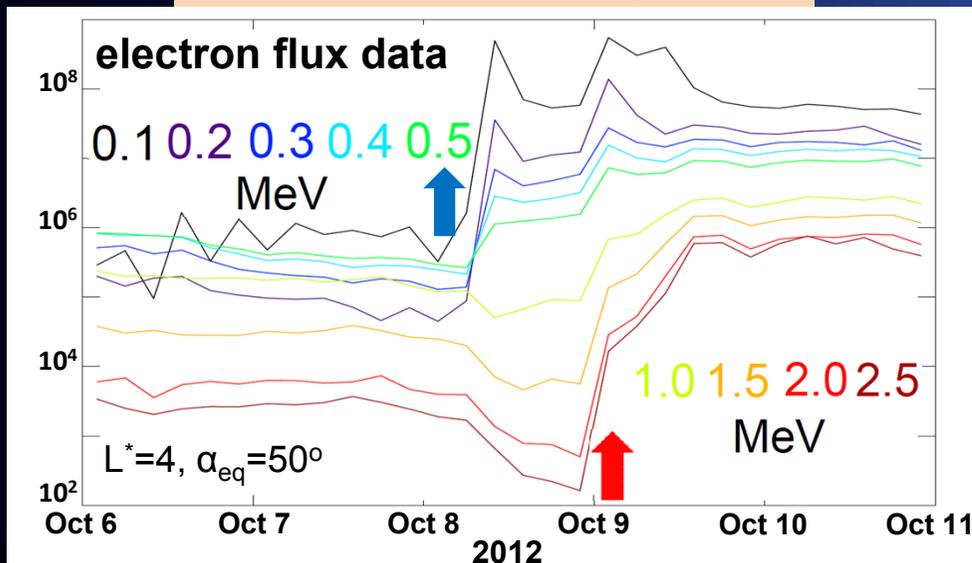
- Need event-specific inputs for very strong events.
- [Tu et al., GRL 2014]: DREAM3D
 - Model the strong enhancement during the Oct 2012 storm.
 - Event-specific chorus waves



3D Diffusion Model

- Need event-specific inputs for very strong events.
- [Tu et al., GRL 2014]: DREAM3D
 - Model the strong enhancement during the Oct 2012 storm.
 - Event-specific chorus waves
 - Event-specific seed electrons

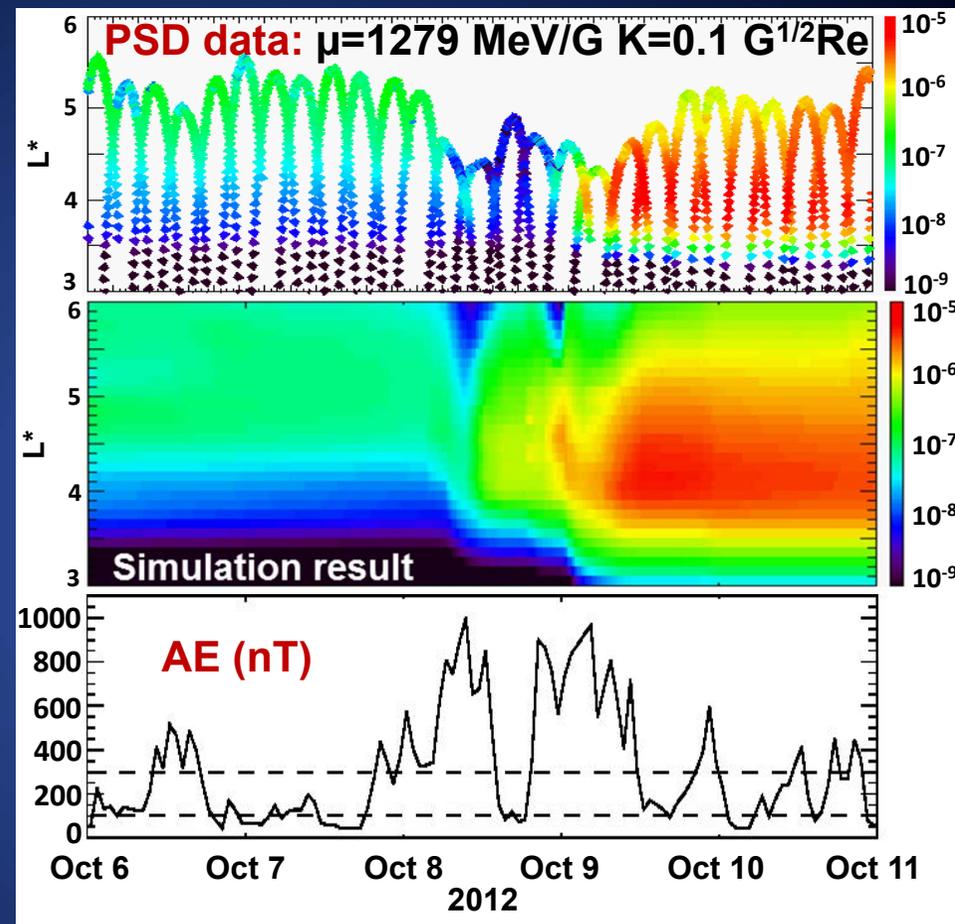
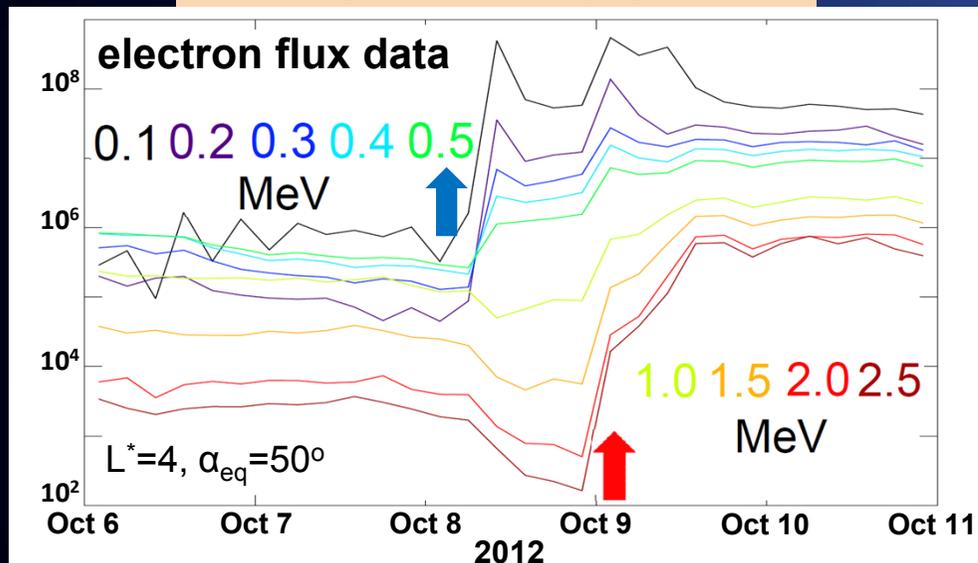
Event-specific seed population



3D Diffusion Model

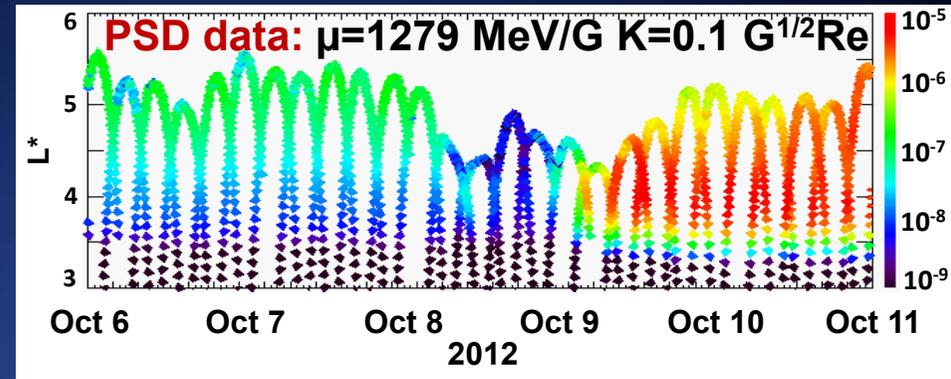
- Need event-specific inputs for very strong events.
- [Tu et al., GRL 2014]: DREAM3D
 - Model the strong enhancement during the Oct 2012 storm.
 - Event-specific chorus waves
 - Event-specific seed electrons

Event-specific seed population



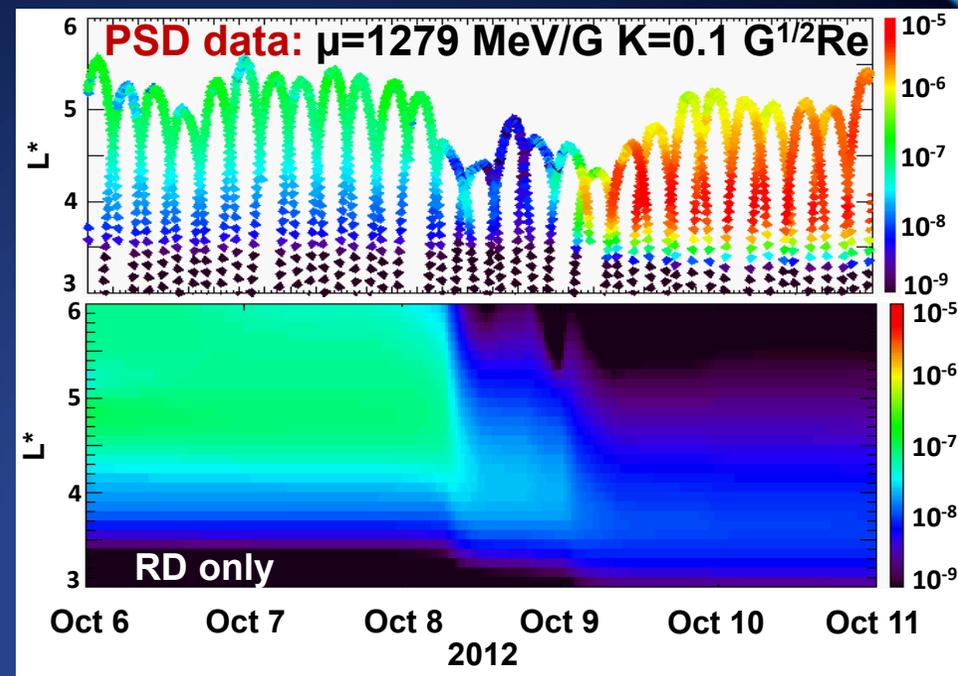
3D Model: Co-operative Physics

- **Old:** Is radial diffusion the **dominant acceleration mechanism** or local acceleration?
- **New:** How do RD, local heating, pitch angle scattering **work together** to produce the observed acceleration and loss of RB electrons?



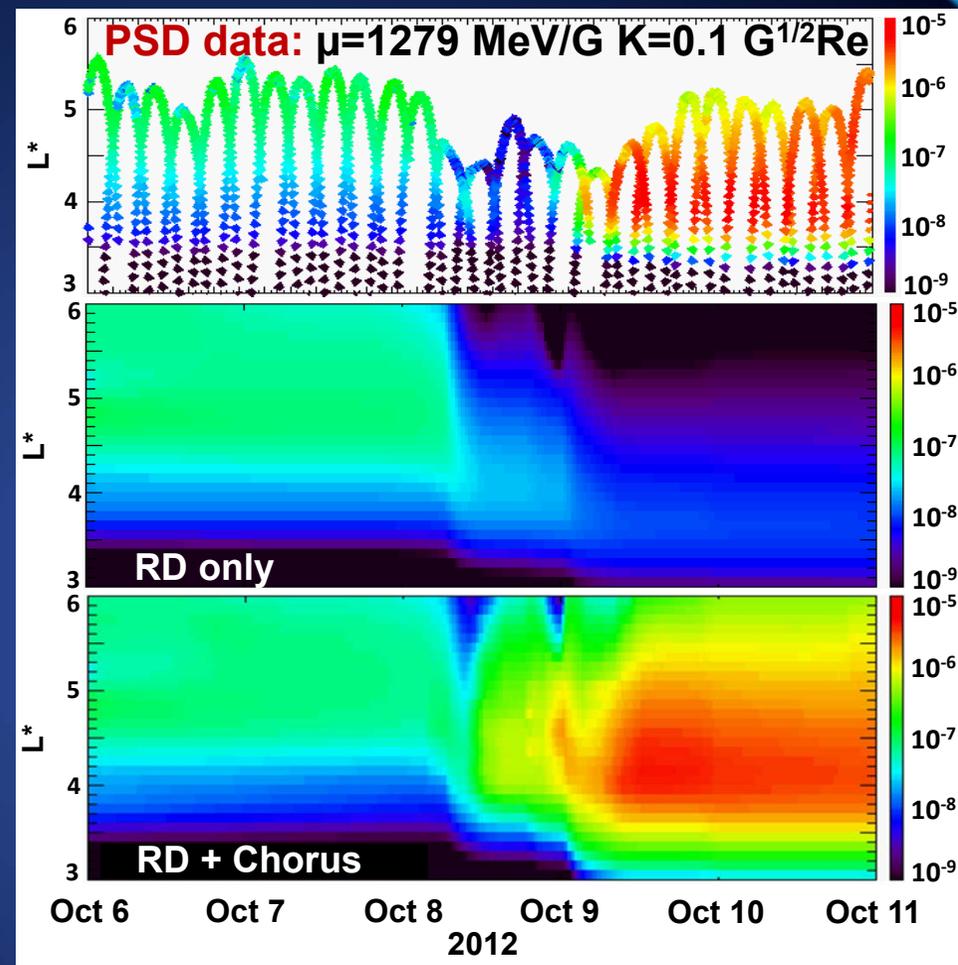
3D Model: Co-operative Physics

- **Old:** Is radial diffusion the **dominant acceleration mechanism** or local acceleration?
- **New:** How do RD, local heating, pitch angle scattering **work together** to produce the observed acceleration and loss of RB electrons?



3D Model: Co-operative Physics

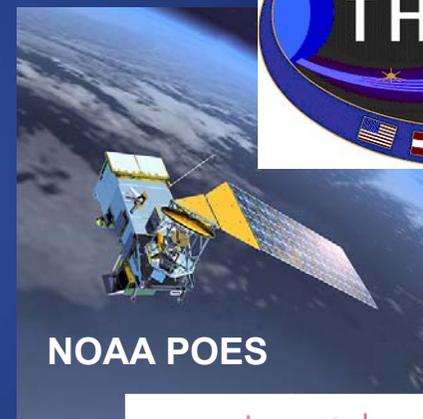
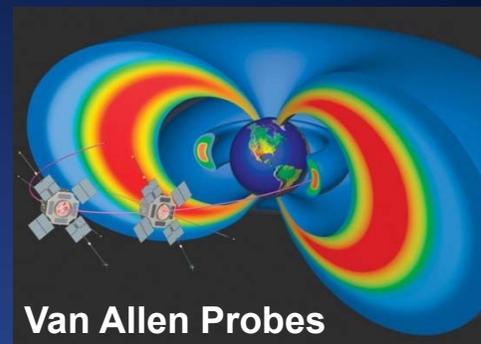
- **Old:** Is radial diffusion the **dominant acceleration mechanism** or local acceleration?
- **New:** How do RD, local heating, pitch angle scattering **work together** to produce the observed acceleration and loss of RB electrons?



- **3D model implies loss mechanisms:** outward RD + precipitation

Advance in RB Models

- **Great variety!**
 - Fokker-Planck diffusion models
 - Convection-Diffusion models
 - Test particles codes
 - PIC & Hybrid codes
- **Advances in modeling techniques**
 - E.g., 1D diffusion \rightarrow 2D diffusion \rightarrow 3D \rightarrow 4D
- **Advances in model inputs:**
 - E.g., chorus wave model:
Static \rightarrow Dynamic but empirical \rightarrow Event-specific
 - **Made possible by the extensive measurements from multiple missions!**



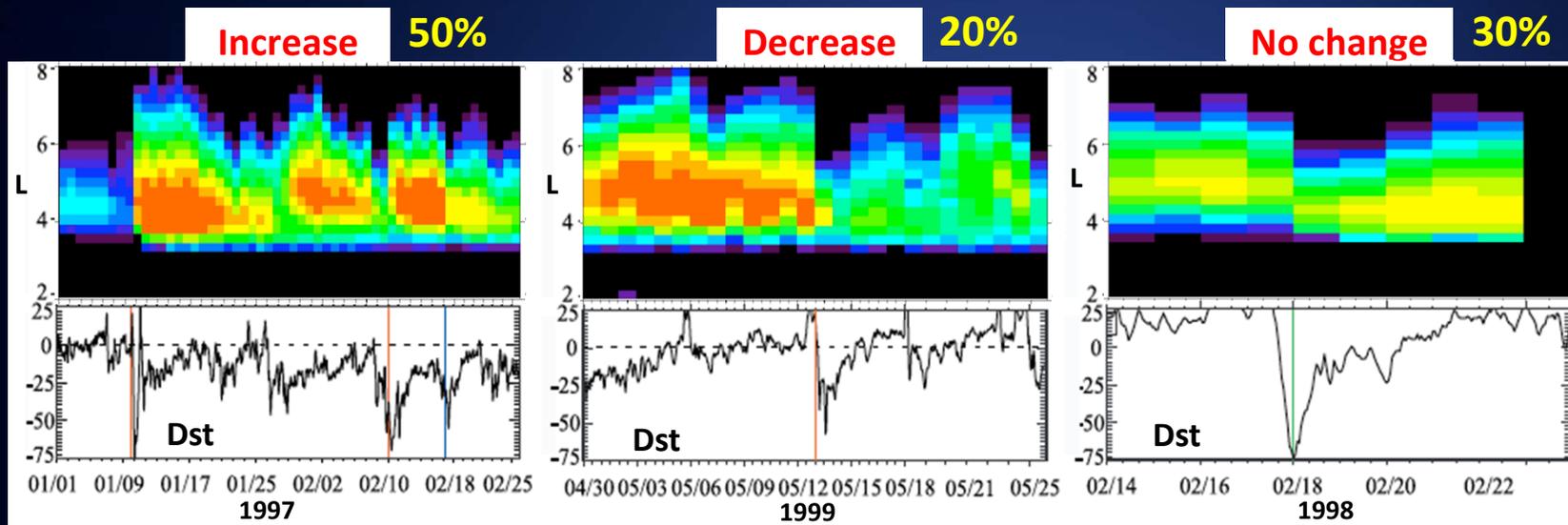
Outline

- Introduction of Radiation Belt Dynamics: Sources and Losses
- Advances in Quantitative RB Modeling: Models and Inputs
- Challenges and Opportunities

Reproduce the Storm Responses

- Complex storm responses of RB electrons:

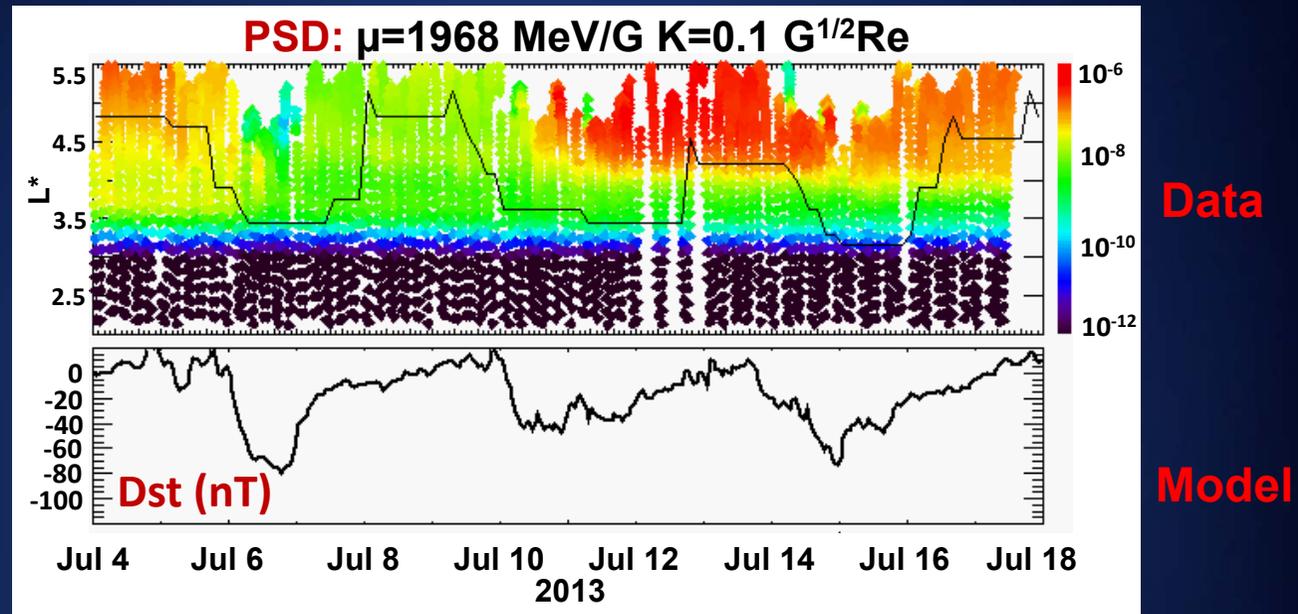
POLAR/HIST 1.2-2.4 MeV electron flux



[Reeves et al., GRL 2003]

Reproduce the Storm Responses

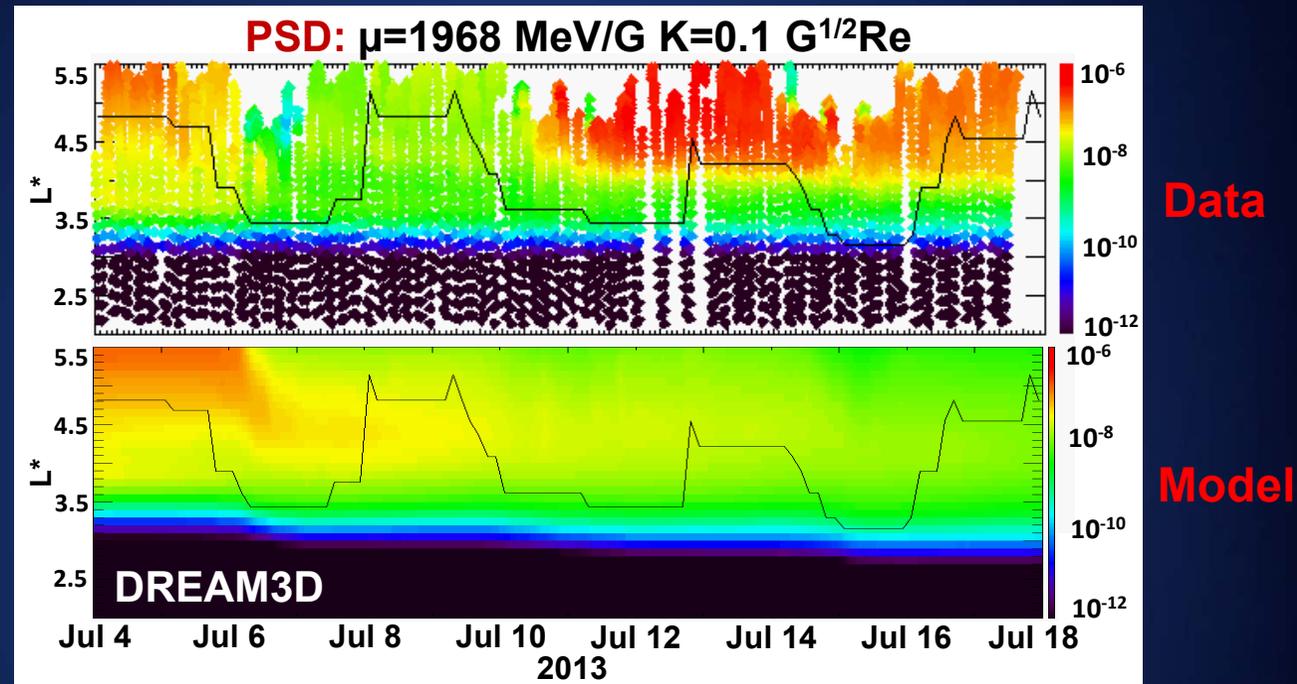
- Complex storm responses of RB electrons:



Reproduce the Storm Responses

- Complex storm responses of RB electrons:

“You have modeled one storm, you have modeled one storm”?



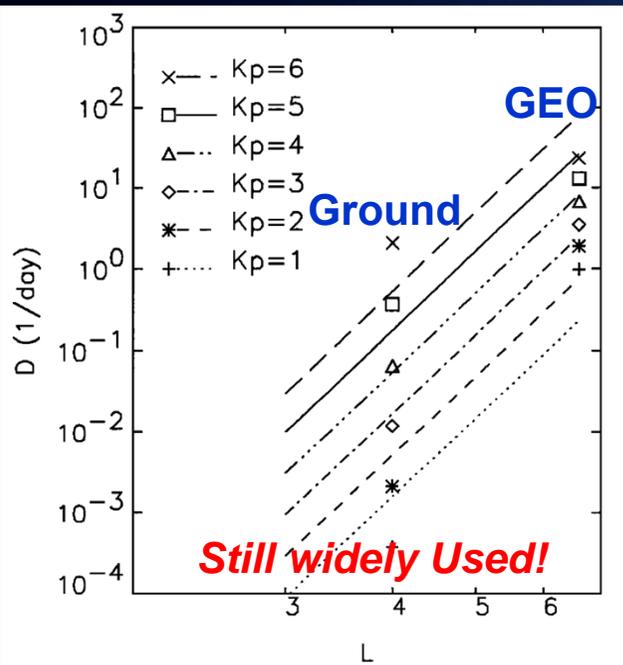
- **Challenge:** Can we model the complex storm variability and, more importantly, predict the various storm responses?
- **Opportunities:** Improve the model inputs & Include new physics.

Better Model Inputs

- Current RB models are mature enough, but **need better and more accurate model inputs.**
- E.g., for diffusion-type models:
 - **Radial diffusion coefficient D_{LL}**

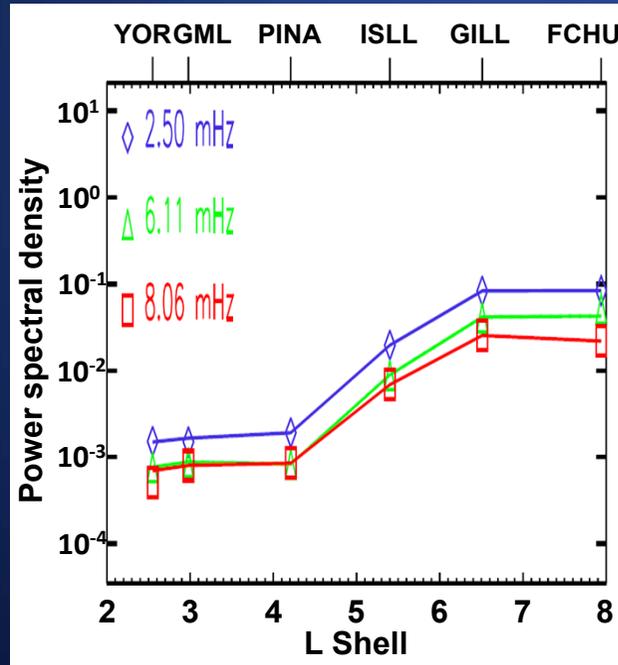
$D_{LL}(Kp)$: 2 points

[Brautigam and Albert, JGR 2000]



$D_{LL}(Kp)$ [Ozeke et al., JGR 2012]

Multiple ground magnetometers



Event-specific D_{LL}

Global ULF waves from:

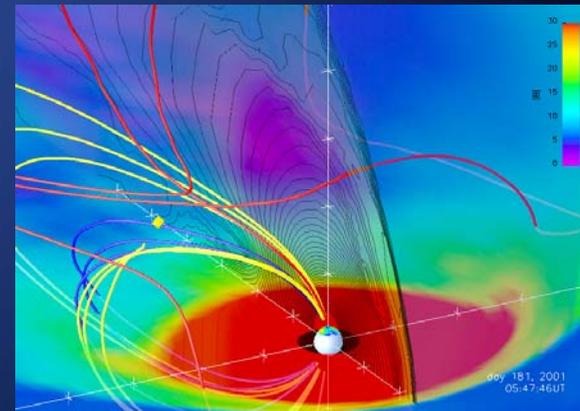
Van Allen Probes

+

Ground magnetometers

+

Global MHD fields



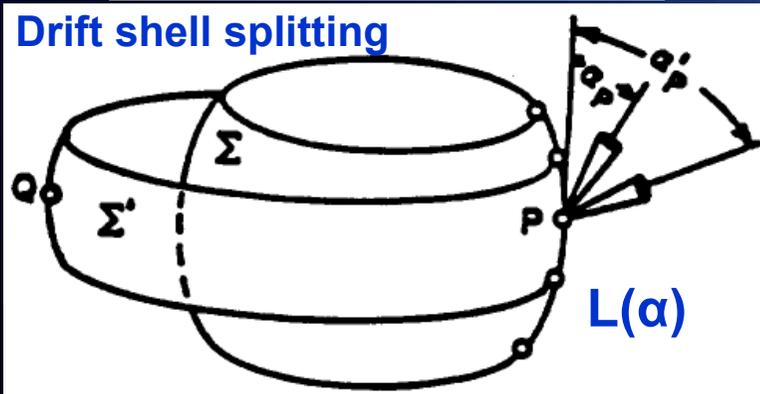
Explore New Physics

- Important to study new physics and quantify its importance.
- E.g., drift shell splitting → anomalous diffusion [O'Brien, GRL 2014]
- Effects of drift shell bifurcation
- Nonlinear, non-resonant wave particle interactions

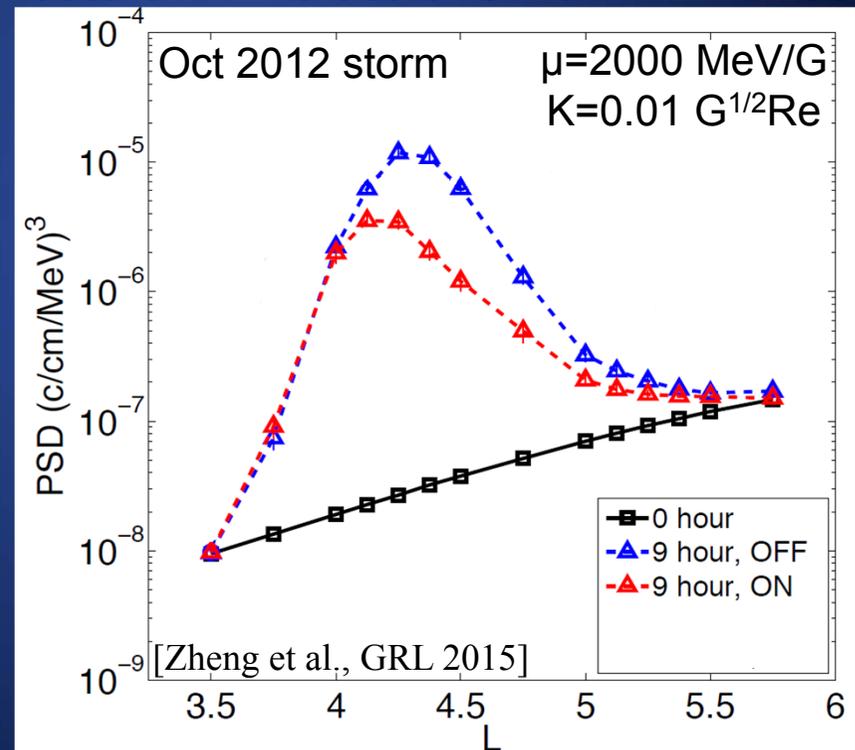
Full diffusion tensor

$$\begin{pmatrix} D_{\mu\mu} & D_{\mu K} & D_{\mu L} \\ D_{K\mu} & D_{KK} & D_{KL} \\ D_{L\mu} & D_{LK} & D_{LL} \end{pmatrix}$$

Drift shell splitting



Effects on RB electrons from REM model



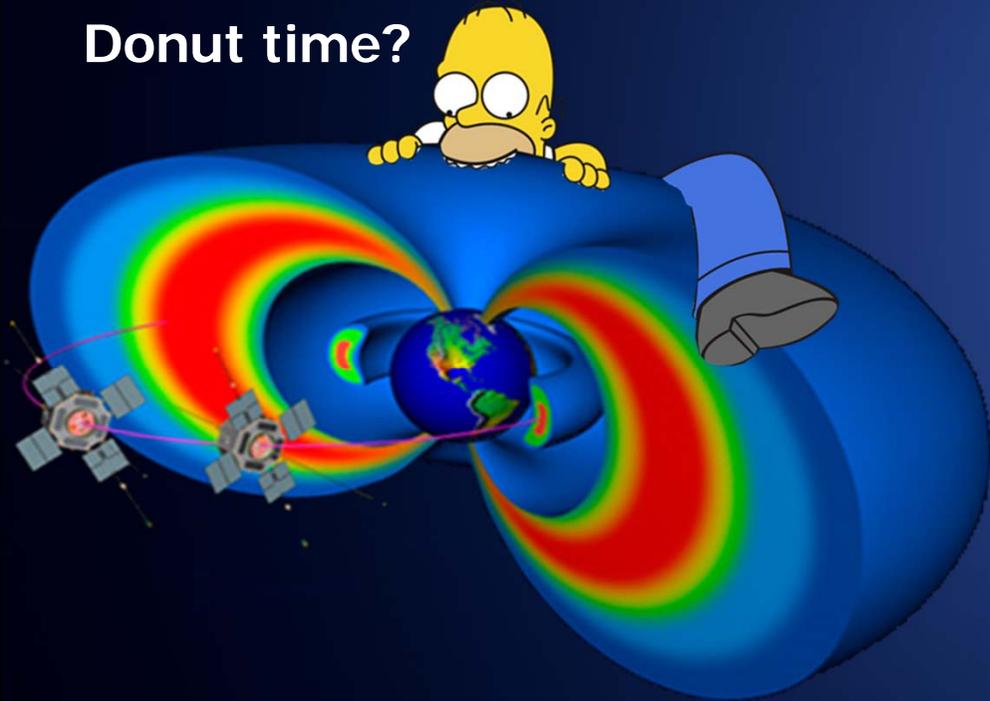
GEM FG: QARBUM (Pronounce: "CHARM")

- Focus Group of “Quantitative Assessment of Radiation Belt Modeling” (2014-2018)
- Co-chairs: Jay Albert, Wen Li, Steve Morley, Weichao Tu
- **Goals:**
 - Bring together the current **state-of-the-art RB models and new physics**.
 - Develop **event-specific and global RB model inputs** (waves, plasma, seed population, magnetic fields).
 - Combine all these components to achieve a **quantitative assessment of the RB modeling** by validating against real-time measurements.
- **Activities:**
 - A review of current RB models and required inputs (last year).
 - “**RB buildup**” and “**RB dropout**” **Challenges** (starting this year).
 - Joint activities with other FGs.

Summary and Conclusions

Thank you!

Donut time?



- Radiation belt is a very dynamic and complicated system due to the delicate balance between source, transport, and loss.
- Great progress has been made in quantitative modeling of radiation belt dynamics:
 - Advances in state-of-the-art models
 - Advances in model inputs
- Challenges on reproducing the complex storm responses, improving model inputs, and exploring new physics.