A Novel Technique for Rapid L* Calculation

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The Scientific Magnetic Mapping and Techniques Focus Group

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Motivation

- L* calculation is essential in radiation belt modeling.
- It is currently computationally expensive to compute L*.
- We propose a new rapid method that calculates L* from first principles.
  - Calculating L* at 181x181 (32761) grid is done in about 2 min.

L^* is...

- Roederer's generalized L value:
  - $L^* = 2\pi k_0 / \Phi R_E$, $\Phi = \int \mathbf{B} \cdot d\mathbf{A} = \oint \mathbf{A} \cdot d\mathbf{l}$
  - Commonly used for quantifying radial transport in the radiation belts.
- Flux integration should be performed <u>along a closed curve in guiding drift shell</u>.

Closed Curve in Energy Space

Total energy:

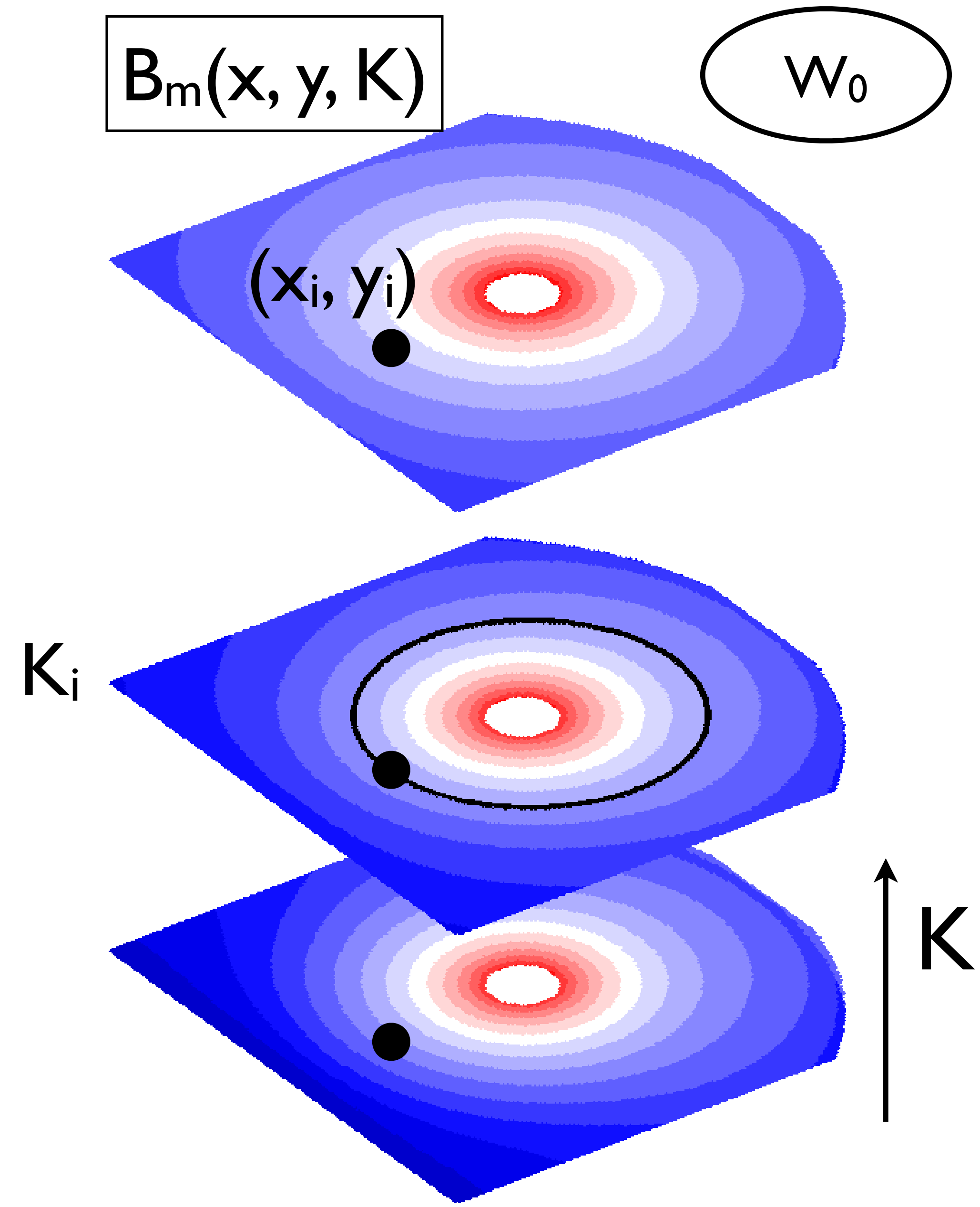
$$W = PE + KE = qU + \mu_m B_m$$

– $B_m(K)$: magnetic field magnitude at mirror point as function of K.

– K: modified longitudinal invariant

$$K = \oint \sqrt{(B_m - B(s))}\, ds$$

Under conservation of energy ~~and first two invariants~~, particle trajectory is iso-energy contours on a mirror point.



References:

"(U, B, K) Coordinates: A natural system for studying magnetospheric convection," Whipple, 1978

"Particle tracing in the magnetosphere: new algorithms and results," Sheldon and Gaffey, 1993

Comparison with LANL* and IRBEM: Input Setup

Table 1. Input parameters for the neural network `LANLstar`.

Table from Koller and Zaharia [2010]

Number	Parameter	Description
1	t_Y	Integer number representing the year = 1996
2	t_{DOY}	Day of the year (int) = 6
3	t_{UT}	Universal Time in units of hours (float) = 1.24 (01h 14m 34s)
4	Dst	Disturbance storm time index (nT) = 7.78
5	p_{sw}	Solar wind dynamic pressure (nPa) = 4.10
6	B_y	Y component of the IMF field (nT) = 3.72
7	B_z	Z component of the IMF field (nT) = -.13
8–13	W_{1-6}	See Tsyganenko and Sitnov (2005) = [.12, .25, .09, .05, .23, 1.05]
14	L_m	McIllwain value; Roederer (1970) = From IRBEM and UBK
15	B_{mirr}	Magnetic field strength at mirror point (nT) = From IRBEM and UBK
16	α_{loc}	Local pitch angle (deg) = [90 60 30 10]°
17	r_{GSM}	Radial coordinate in GSM system (R_E) = [9 8 7 6 5 4 3]
18	θ_{GSM}	Latitudinal coordinate in GSM (deg) = 0°
19	φ_{GSM}	Longitudinal coordinate in GSM (deg) = 180°
20	Vsw	Solar wind velocity (km/s) = 400.10
21	Φ	Electric potential (kV/R_E) = constant (i.e. $W' = \mu_m B_m$)

Tile≈-30°

Comparison with IRBEM: Input Setup

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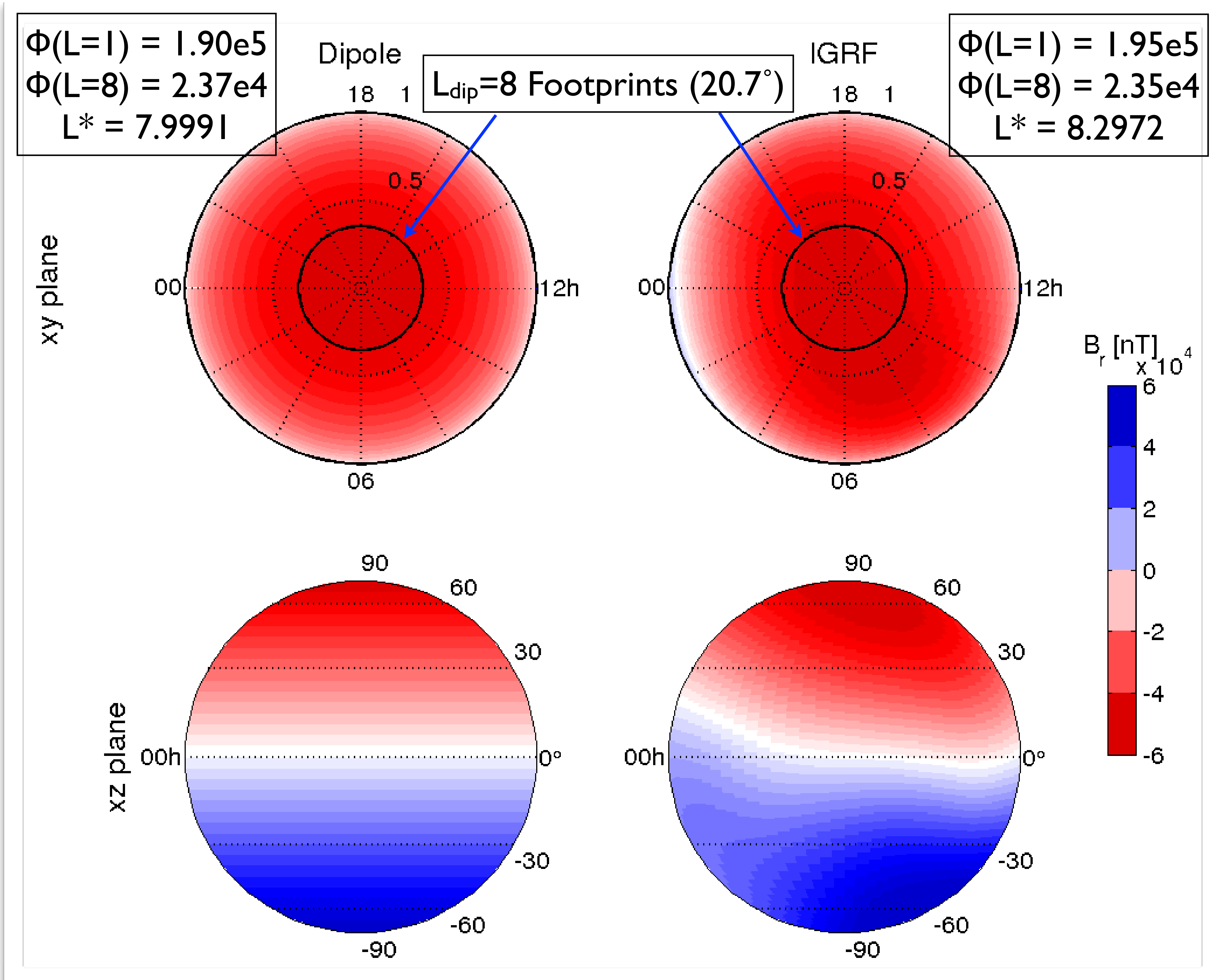
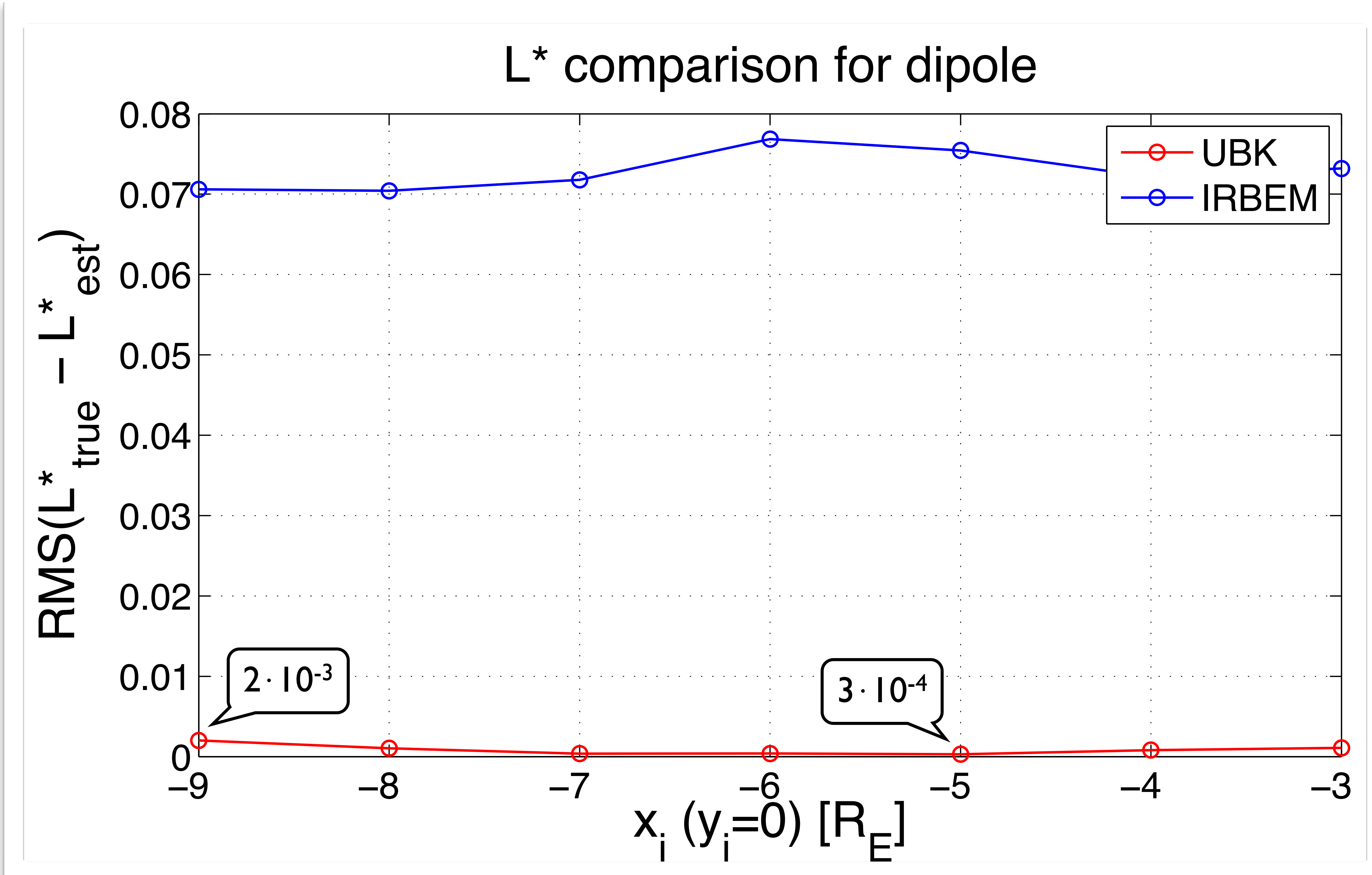
Dipole vs. IGRF

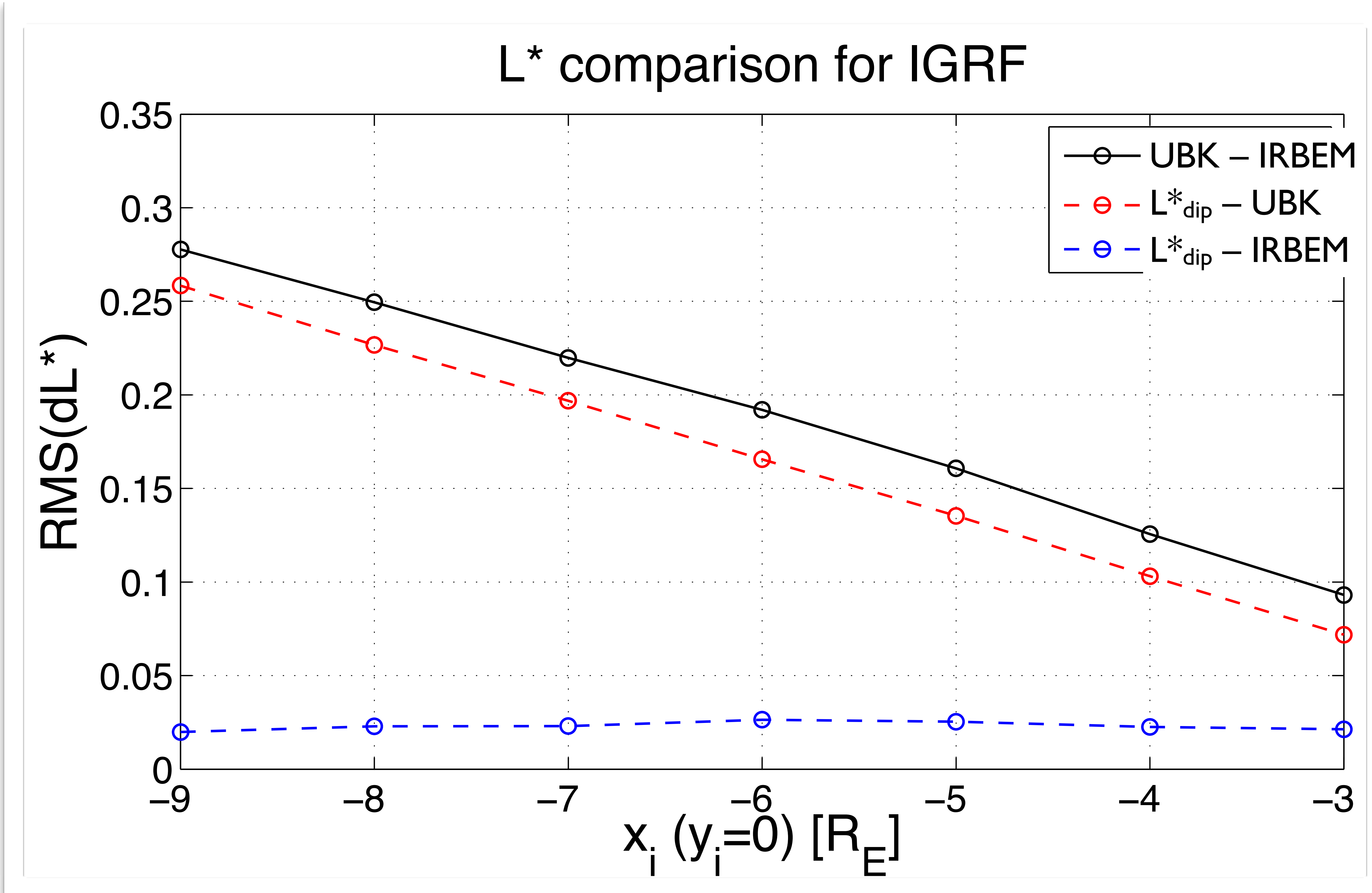


L* (U=0): Comparison with IRBEM: Dipole Only



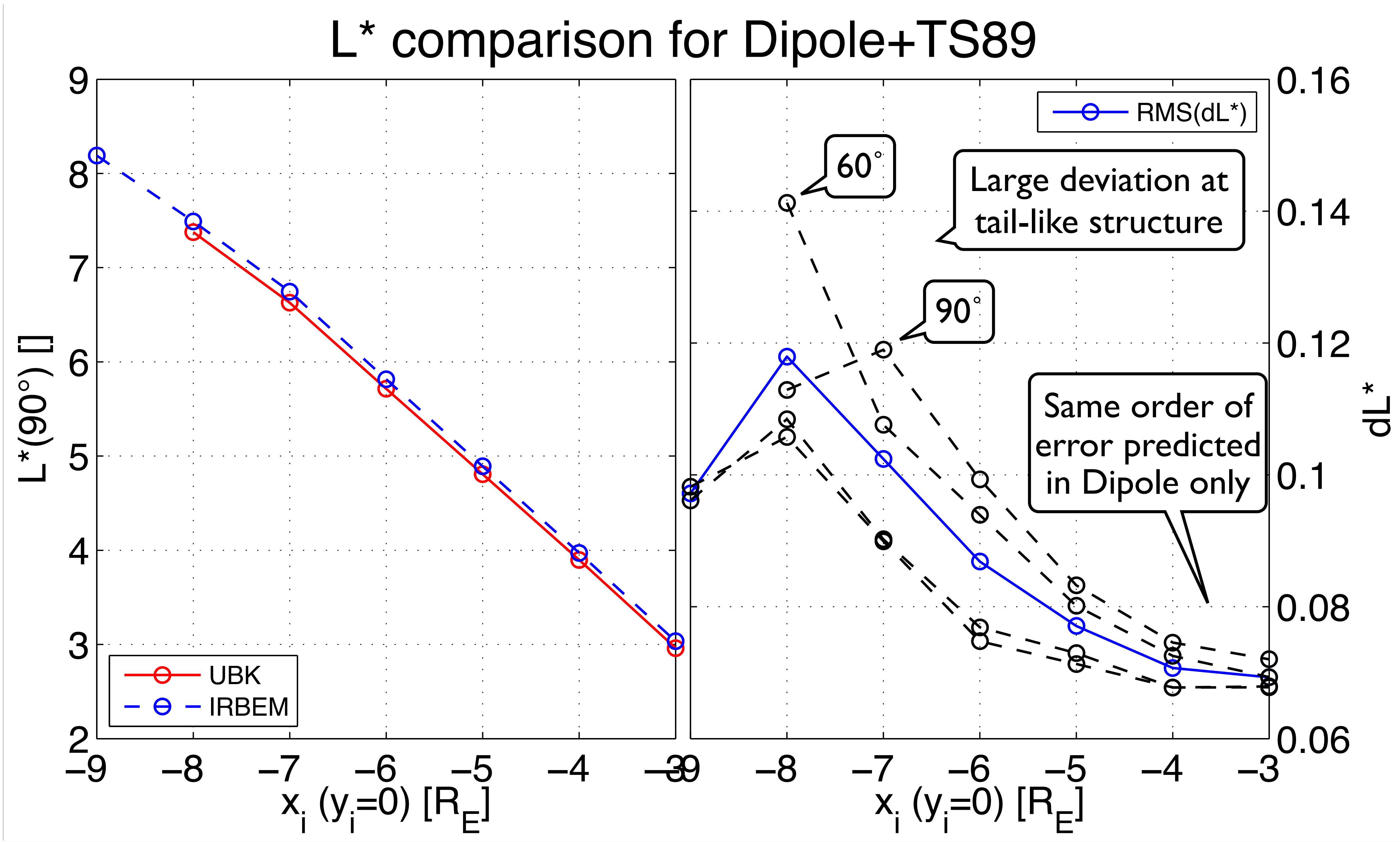
$err = \sqrt{(\sum_{PA}(dL^*)^2/n_{PA})}$

L* (U=0): Comparison with IRBEM: IGRF Only

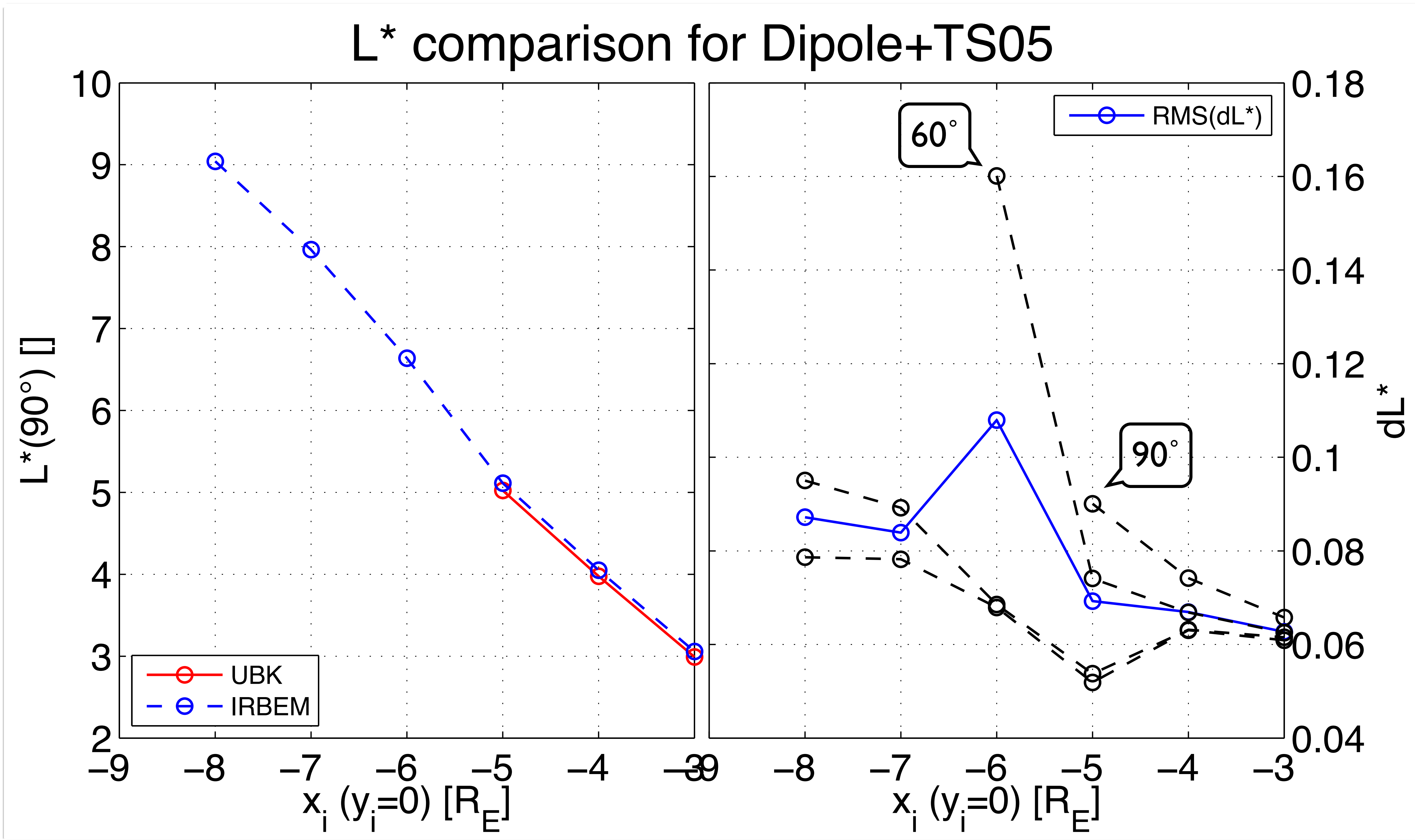


$$L^*_{dip} = \oint_{surf} A_{dip} \cdot dl \;/\; \oint_{orb} A_{dip} \cdot dl$$

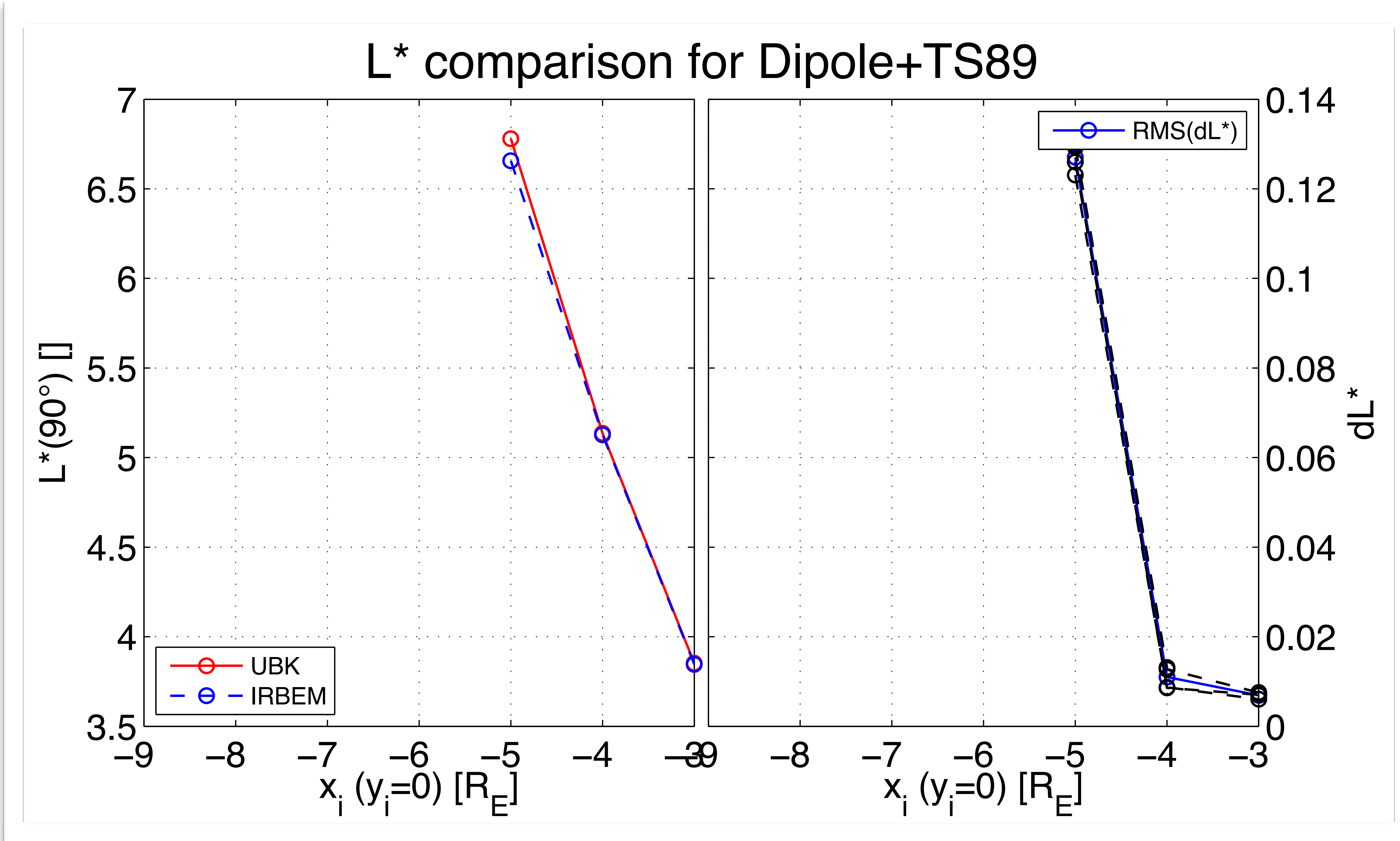
Comparison with IRBEM: Dipole+TS89 [SM]



Comparison with IRBEM: Dipole+TS05 [SM]



Comparison with IRBEM: Dipole+TS89 [GSM]



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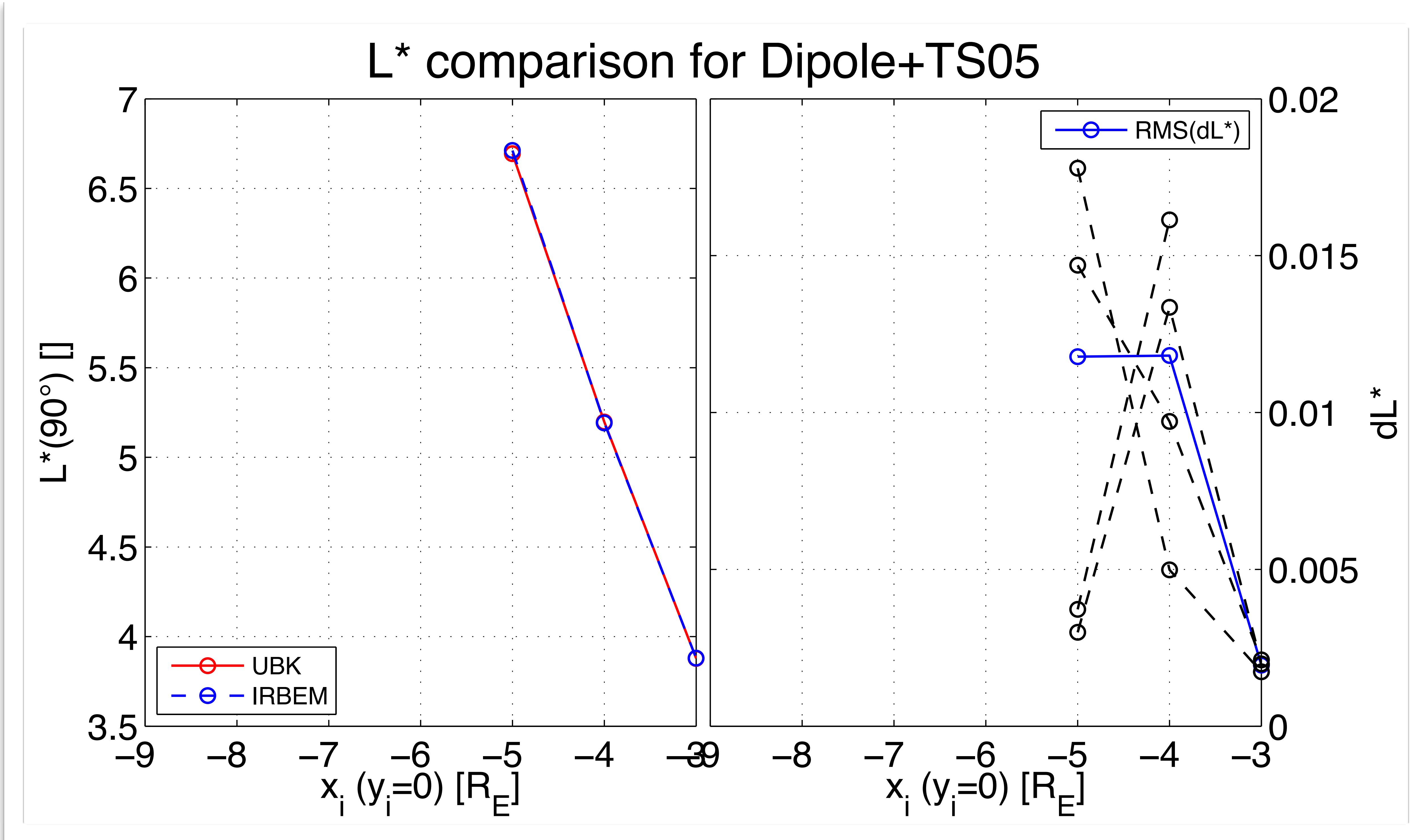
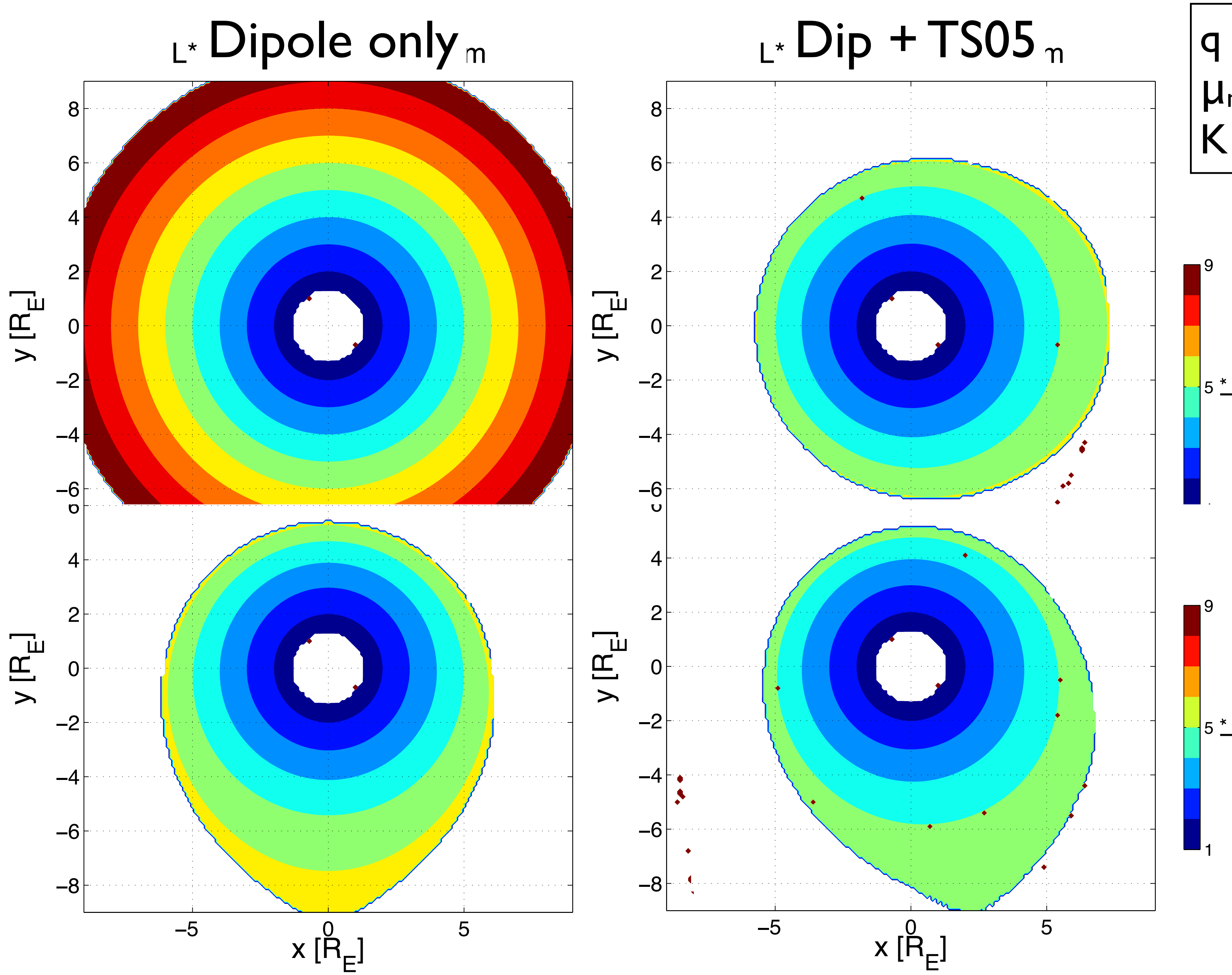


Speed & L* Map

MATLAB		Fortran	Fortran
(x, y)	UBK	IRBEM (DIP only)	LANL*
181x181	~ 2 min	> 24+ min	< .5 sec



$q = e$
μ_m = 1 keV/nT
$K = 0.1\ \sqrt{nT}\ R_E$

Φ = const.

$\Phi = -A/r - B\ r \sin(\varphi)$, where A=92 kV/$R_E$, B=10 kV.

Summary

1. Trajectory calculation in energy space is preferable because
   – No error accumulation due to numerical integration (only errors are from numerical interpolation),
   – Strict conservation of energy and 2nd invariant (accurate contour and perfectly closed if closed),
   – Fast computation: contour calculation is rapid (ex. contour plot) and independent of initial value of a particle, and
   – Based on first principles only.

2. Preparation step is required to compute $B_m(K)$ and the time depends on the field model (≈10 minutes for TS05).

3. The difference (dL*) is generally less than 0.1 and this method outperforms, at least in a dipole-like magnetic field, the previous method based on Lagrangian approach.

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